# Some Applications of Discrete One Parameter Singular Perturbation Method

G. Kishore Babu * and M. S. Krishnarayalu **

* Department of Electrical Engineering, PSCMR Engineering College, Vijayawada, A. P. 520002, India

** Department of Electrical and Electronics Engineering, V.R Siddhartha Engineering College,Vijayawada, A. P. 520007, India.

*Abstract*- **Discrete One Parameter Singular Perturbation Method is extended up to second-order approximation. A third order discrete power system model with two time scales is considered. It is modeled as a one parameter singularly perturbed system. Then Initial Value Problem (IVP) and Boundary Value Problems (BVP) are studied using this Singular Perturbation Method (SPM). SPM consists of an outer series solution and one boundary layer correction (BLC) solution. A boundary layer correction is required to recover the initial conditions lost in the process of degeneration and to improve the solution. SPM is carried out up to second-order approximate solution for the load frequency control (LFC) model of a single area power system for both IVP and BVP. The results are compared with the exact solution. The results substantiate the application.**

***Keywords- Time Scales; Discrete One Parameter System; Singular Perturbation Method; Boundary Layer Correction; Initial Value Problem; Boundary Value Problem***

## I. INTRODUCTION

The dynamics of many continuous-time and discrete-time systems is described by high order differential and difference equations respectively. The presence of small parameters such as time constants, masses, moments of inertia, inductances and capacitances is the source for increased order of the system. The solution of these high order stiff systems poses a problem and requires special numerical methods [21]. SPM alleviates these problems by suppressing the small parameters. More specifically, SPM removes system's stiffness, reduces the order of the system, satisfies the given boundary conditions and gives an approximate solution closer to the exact solution.

A system in which the suppression of a small parameter results in degeneration of the dimension of the system is called a singularly perturbed system. Such a system possesses widely separated groups of eigenvalues exhibiting slow and fast phenomena or time-scale phenomena. Singularly perturbed and time-scale systems are identical [10]. Power system control results in multi-time-scale systems as number of time constants of different magnitudes (governor, turbine, generator & load, amplifier, exciter) are involved [5]. Also multi-time-scale (MTS) systems exhibit the chaotic characteristic [22]. Hence obtaining the exact solution of MTS is difficult and special numerical methods are required [21].

SPM in continuous control systems has matured enough [1-8]. SPM in discrete control systems is being developed [6, 9-19] and its applications are not thoroughly explored. To fill that gap, here a stable third order discrete power system model [20] with two time scales is considered. It is modeled as a one parameter singularly perturbed system. Then IVP & BVP are studied using the SPM extended up to second-order approximation for the LFC model of a single area power system.

## II. DISCRETE ONE-PARAMETER SINGULAR PERTURBATION METHOD (DOPSPM)

The one-parameter and multi-parameter problems in discrete systems are studied extensively [9-19]. Here we overview the DOPSPM up to second-order approximation in state variable form from control viewpoint. Consider a two-time-scale linear time-invariant stable system described by

$$\begin{bmatrix} x_0(k+1) \\ x_1(k+1) \end{bmatrix} = A \begin{bmatrix} x_0(k) \\ \varepsilon x_1(k) \end{bmatrix} + Bu(k) \tag{1a}$$

where

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},\ B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

and state vector $x_j(k) \in \Re^{n_j}$ , j=0, 1. $A_{ij}$ and $B_i$ are matrices of approximate dimensionality; ε is the small positive scalar parameter → 0+ . The control vector $u(k) \in \Re^r$ is independent of the small parameters. With initial conditions

$$x_j(k=0) = x_j(0),\ j = 0,1. \tag{1b}$$

(1) gives a general form of initial value problem (IVP) of singularly perturbed one-parameter discrete control system. When ε is made equal to zero in (1), $x_1(0)$ is lost, i.e. $x_1(0)$ is sacrificed in the process of degeneration. The resulting degenerate subsystem is given by

$$\begin{bmatrix} x_0^0(k+1) \\ x_1^0(k+1) \end{bmatrix} = A \begin{bmatrix} x_0^0(k) \\ 0 \end{bmatrix} + B\,u(k). \tag{2a}$$

It is of reduced order $n_0$ and cannot satisfy all the given initial conditions (1b). Hence the discrete system (1) is in the singularly perturbed form. That is

$$x_0^0(0) = x_0(0);\quad x_1^0(0) \neq x_1(0). \tag{2b}$$

The $n_1$ initial conditions lost in the process of degeneration are recovered by the following SPM. For example, for the IVP of power system model (Table 1), the specified values of initial conditions are $x_{01}(0) = 5$, $x_{02}(0) = 5$ *and* $x_1(0) = 3$. Here $x_{01}$ and $x_{02}$ are slow variables and $x_1$ is fast variable.

The degenerate values are $x_{01}^0(0)$ =5, $x_{02}^0(0) = 5$ and $x_1^0(0) = 0.0335$. Hence the degenerate system, obtained by making ε = 0, is unable to satisfy the initial condition of the fast variable $x_1$. Hence there is a need to recover this initial condition.

*A. Singular perturbation method for IVP*

SPM for discrete systems consists of an outer solution and a BLC solution for one-parameter discrete problems. Outer solution is obtained by suppressing the small parameter ε in (1a). Here boundary layers are formed due to rapid changes in solution caused by stable fast modes during initial time ($x_1(0)$ = 3 to $x_1(1)$=0.3018 in Table 1). As the fast modes are suppressed in outer solution, a BLC solution is required to recover the conditions lost in degeneration and to improve the solution. These concepts become clear when we analyze the data of Table 1 as above. The boundary layer jump for the fast variable $x_1$ at k = 0 is from 3 to 0.0335 ($x_1(0)\ to\ x_1^0(0)$) indicating nonuniform convergence of the exact solution to the degenerate solution.

*a) Outer solution*

We assume double asymptotic power series expansions for outer series solution as

$$x_{vout}(k) = \sum_{i\geq 0}^{q} x_v^i(k)\varepsilon^i, \text{ v=0,1.} \tag{3}$$

where q is the order of the desired approximation. By substituting (3) in (1a) and equating coefficients of like powers of $\varepsilon^i$, we obtain a set of equations. For the zero-order approximation, $\varepsilon^0$, the resulting equation is the same as that given by (2a). For first and second-order approximations,

$$\varepsilon^j : \begin{bmatrix} x_0^j(k+1) \\ x_1^j(k+1) \end{bmatrix} = \text{A} \begin{bmatrix} x_0^j(k) \\ x_0^{j-1}(k) \end{bmatrix}, \text{ j =1,2.} \tag{4}$$

Similar equations can be easily formed for higher-order approximations.

*b) Boundary layer correction solutions*

Boundary layers are formed due to the nonuniform convergence of the exact solution to the degenerate solution. Boundary layers correspond to the rapid region of transition in the exact solution. Now boundary layer corrections have to be added to recover the lost boundary conditions and to improve the degenerate solution. Also boundary layer corrections should ensure that the solution is unique. In order to recover the initial conditions lost in the process of degeneration and to provide the necessary initial data for solving outer equation (5), the following transformations are used.

$$x_{0C}(\text{k}) = \frac{x_0(k)}{\varepsilon^{k+1}} \tag{5a}$$

$$\text{x}_{1C}(\text{k}) = \frac{\text{x}_1(\text{k})}{\varepsilon^{\text{k}}} \tag{5b}$$

where $(x_{0c}, x_{1c})$ refers to the BLC solutions.

Using (5) in (1a) the subsystem for the BLC is given by

$$\begin{bmatrix} \varepsilon x_{0c}(\text{k}+1) \\ x_{1c}(\text{k}+1) \end{bmatrix} = \text{A} \begin{bmatrix} x_{0c}(\text{k}) \\ x_{1c}(\text{k}) \end{bmatrix} \tag{6}$$

Double asymptotic power series expansions are implemented for BLC

$$x_{vc}(\text{k}) = \sum_{i\geq 0}^{q} x_{\text{vc}}^i(k)\,\varepsilon^i, \quad v = 0,1. \tag{7}$$

Substituting (7) in (6) and collecting coefficients gives for the zero-order approximation

$$\varepsilon^0: \begin{bmatrix} 0 \\ x_{1C}^0(k+1) \end{bmatrix} = \text{A} \begin{bmatrix} x_{0C}^0(k) \\ x_{1C}^0(k) \end{bmatrix} \tag{8a}$$

For first and second-order approximations

$$\varepsilon^j : \begin{bmatrix} x_{0C}^{j-1}(k+1) \\ x_{1C}^j(k+1) \end{bmatrix} = \text{A} \begin{bmatrix} x_{0C}^j(k) \\ x_{1C}^j(k) \end{bmatrix}, \text{ j = 1,2.} \tag{8b}$$

*c) Total series solution (TSS)*

The total series solution is given by the outer solution (3) and the BLC solution (7) as

$$x_0^q(k) = \sum_{i\geq 0}^{q} [x_0^i(k) + \varepsilon^{k+1} x_{0C}^i(k)]\varepsilon^i \tag{9a}$$

$$x_1^q(k) = \sum_{i\geq 0}^{q} [x_1^i(k) + \varepsilon^k x_{1C}^i(k)]\varepsilon^i \tag{9b}$$

where q is the order of the desired approximation.

*d) Initial conditions*

The determination of the necessary initial conditions for the solution of the outer equations (2a) and (4) and the BLC equations (8) is a vital step in singular perturbation analysis. These are determined based on the fact that the total series solution (9) consisting of outer and BLC solutions should satisfy the given initial conditions. Then the following relations are obtained. For zero-order approximation

$$\varepsilon^0:\ x_0^0(0) = x_0(0) \tag{10a}$$

$$\varepsilon^0: x_{1C}^0(0) = x_1(0) - x_1^0(0) \tag{10b}$$

For first and second-order approximations ( $\varepsilon^j$ )

$$x_0^j(0) = -x_{0C}^{j-1}(0) \tag{10c}$$

$$x_{1C}^j(0) = -x_1^j(0),\ j = 1,2 \tag{10d}$$

For IVP shown in Table 1, for zero-order approximation

$$x_0^0(0) = x_0(0) = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

$$x_{1C}^0(0) = x_1(0) - x_1^0(0) = 3 - 0.0335 = 2.9665$$

This selection will satisfy the specified initial conditions of the system.

*e) Algorithm*

For a particular order of approximation, first find the outer solution and then obtain the BLC solution. For both these computations we have to use the initial conditions (10). Finally using (9) compute the TSS for that order of approximation. First zero-order solution is to be found; then first-order, second-order and so on.

*B. Singular perturbation method for BVP*

We are considering a stable system with fast and slow modes. For stable fast modes the boundary layers occur at initial point (k=0). Hence $x_1(k)$ will be specified at initial point as in IVP. Slow modes represented by $x_0(k)$ may be specified at any point. We take that $x_0(k)$ is specified at the final point (k=N) as $x_0(k=N) = x_0(N)$; N is a positive integer. Then the problem at hand is a Two Point Boundary Value Problem (TPBVP). SPM for this problem is same as that IVP except for selection of final conditions of $x_0(k)$. For this TPBVP the final conditions demanded by TSS (9) are furnished as

$$\varepsilon^0:\ x_0^0(N) = x_0(\mathrm{N}) \tag{11a}$$

$$\varepsilon^j:\ x_0^j(N) = 0\ ,\ \mathrm{j}=1,2. \tag{11b}$$

*C. Applications of DOPSPM*

*a) IVP*

Consider the third order single area power system model used for LFC sampled with 0.2s [20]. The resulting system is given by

$$\begin{bmatrix} x_{01}(k+1) \\ x_{02}(k+1) \\ x_1(k+1) \end{bmatrix} = \begin{bmatrix} 0.772 & 0.037 & -0.017 \\ 0.085 & 0.720 & 0.0406 \\ -0.244 & 0.145 & 0.0306 \end{bmatrix} \begin{bmatrix} x_{01}(k) \\ x_{02}(k) \\ x_1(k) \end{bmatrix} +$$

$$\begin{bmatrix} -0.144 \\ 0.074 \\ 0.705 \end{bmatrix} \mathrm{u(k)} \tag{12a}$$

Here $x_0 = \begin{bmatrix} x_{01} \\ x_{02} \end{bmatrix}$ and u(k) is unit step function. The Eigen spectrum of this system

(0.8078, 0.6990, 0.0151)

clearly indicates two-time-scale nature with two slow modes and one fast mode. Hence it is represented as a one-parameter system as shown below.

$$\begin{bmatrix} x_{01}(k+1) \\ x_{02}(k+1) \\ x_1(k+1) \end{bmatrix} = \begin{bmatrix} 0.772 & 0.037 & -0.088 \\ 0.085 & 0.720 & 0.203 \\ -0.244 & 0.145 & 0.153 \end{bmatrix} \begin{bmatrix} x_{01}(k) \\ x_{02}(k) \\ \varepsilon x_1(k) \end{bmatrix} +$$

$$\begin{bmatrix} -0.144 \\ 0.074 \\ 0.705 \end{bmatrix} \mathrm{u(k)} \tag{12b}$$

where $\varepsilon = 0.2$.

The initial conditions are given as

$$x_{01}(0) = 5;\quad x_{02}(0) = 5;\quad x_1(0) = 3.$$

This is an IVP where all the conditions are specified at initial point (k=0). When the small parameter ε is suppressed, the resulting degenerate system (2a) yields the following values

$$x_{01}(0) = 5,\ x_{02}(0) = 5 \text{ and } x_1(0) = 0.0335.$$

Hence the degenerate system fails to satisfy the specified value of $x_1(0) = 3$.

*b) BVP*

The boundary conditions are given as

$$x_{01}(10) = 1;\quad x_{02}(10) = 1;\quad x_1(0) = 5.$$

This is TPBVP as $x_{01}$ and $x_{02}$ are specified at k =10 and $x_1$ is specified at initial point (k=0). When the small parameter ε is suppressed, the resulting degenerate system (2a) yields the following values

$$x_{01}(10) = 1,\ x_{02}(10) = 1 \text{ and } x_1(0) = -8.0464.$$

Hence the degenerate system fails to satisfy the specified value of $x_1(0) = 5$.

The IVP and BVP are solved using the SPM given in Section 2 to recover the initial/boundary condition lost in the process of degeneration and further improve the approximate solution. The solutions for zero, first, second-order approximations are obtained and compared with the exact solution as shown in the Tables 1 and 2 for IVP and BVP respectively. From these tables we observe that

(i). The degenerate solution, obtained by making $\varepsilon$ equal to zero in (12a), is unable to satisfy the initial conditions $x_1(0)$.

(ii).The zero-order solution, obtained from (9), incorporates BLCs and hence it recovers the initial conditions $x_1(0)$.

Thereafter, i.e., $k \geq 1$, it remains equal to the degenerate solution.

(iii). Boundary layer (region of rapid transition) is formed at k = 0 for $x_1$ (the change from exact to degenerate solution is 3 to 0.0335 in IVP and 5 to -8.0464 for BVP).

(iv).The first-order solution improves the zero-order solution and is closer to the exact solution. The second-order solution improves the first-order solution and is much closer to the exact solution.

TABLE 1: COMPARISON OF VARIOUS SERIES SOLUTIONS WITH THE EXACT SOLUTION FOR IVP

| x(k) | Degenerate Solution | Zero- order Solution | First- order Solution | Second-order Solution | Exact Solution |
|---|---|---|---|---|---|
| $x_{01}(0)$ | 5 | 5 | 5 | 5 | 5 |
| $x_{02}(0)$ | 5 | 5 | 5 | 5 | 5 |
| $x_1(0)$ | 0.0335 | 3 | 3 | 3 | 3 |
| | | | | | |
| $x_{01}(1)$ | 3.9010 | 3.9010 | 3.8483 | 3.8482 | 3.8482 |
| $x_{02}(1)$ | 4.0990 | 4.0990 | 4.1024 | 4.2151 | 4.2208 |
| $x_1(1)$ | 0.2100 | 0.2100 | 0.8222 | 0.5390 | 0.3018 |

| | | | | | |
|---|---|---|---|---|---|
| | | | | | |
| $x_{01}$ (2) | 3.0192 | 3.0192 | 2.9751 | 2.9769 | 2.9777 |
| $x_{02}(2)$ | 3.3169 | 3.3169 | 3.3234 | 3.3996 | 3.4523 |
| $x_1(2)$ | 0.3475 | 0.3475 | 0.3673 | 0.3791 | 0.3873 |
| | | | | | |
| $x_{01}$ (3) | 2.3111 | 2.3111 | 2.2711 | 2.2735 | 2.2757 |
| $x_{02}(3)$ | 2.7476 | 2.7476 | 2.7626 | 2.8062 | 2.8285 |
| $x_1(3)$ | 0.4551 | 0.4551 | 0.4785 | 0.4891 | 0.4909 |
| | | | | | |
| $x_{01}$ (4) | 1.7416 | 1.7416 | 1.7066 | 1.7079 | 1.7088 |
| $x_{02}(4)$ | 2.2487 | 2.2487 | 2.2708 | 2.3105 | 2.3239 |
| $x_1(4)$ | 0.5395 | 0.5395 | 0.5652 | 0.5696 | 0.5749 |
| | | | | | |
| $x_{01}$ (5) | 1.2839 | 1.2839 | 1.2458 | 1.2499 | 1.2511 |
| $x_{02}(5)$ | 1.8411 | 1.8411 | 1.8782 | 1.9013 | 1.9155 |
| $x_1(5)$ | 0.6061 | 0.6061 | 0.6356 | 0.6399 | 0.6428 |
| | | | | | |
| $x_{01}$ (6) | 0.9153 | 0.9153 | 0.8766 | 0.8795 | 0.8804 |
| $x_{02}(6)$ | 1.5087 | 1.5087 | 1.5568 | 1.5783 | 1.5858 |
| $x_1(6)$ | 0.6587 | 0.6587 | 0.6919 | 0.6951 | 0.6972 |
| | | | | | |
| $x_{01}$ (7) | 0.6184 | 0.6184 | 0.5787 | 0.5800 | 0.5829 |
| $x_{02}(7)$ | 1.2381 | 1.2381 | 1.2960 | 1.3015 | 1.3590 |
| $x_1(7)$ | 0.7004 | 0.7004 | 0.7369 | 0.7395 | 0.7412 |
| | | | | | |
| $x_{01}$ (8) | 0.3792 | 0.3792 | 0.3354 | 0.3403 | 0.3417 |
| $x_{02}(8)$ | 1.0180 | 1.0180 | 1.0847 | 1.1009 | 1.1033 |
| $x_1(8)$ | 0.7336 | 0.7336 | 0.7730 | 0.7749 | 0.7767 |
| | | | | | |
| $x_{01}$ (9) | 0.1864 | 0.1864 | 0.1444 | 0.1465 | 0.1470 |
| $x_{02}(9)$ | 0.8392 | 0.8392 | 0.9135 | 0.9253 | 0.9190 |
| $x_1(9)$ | 0.7001 | 0.7001 | 0.8021 | 0.8049 | 0.8054 |
| | | | | | |
| $x_{01}(10)$ | 0.0310 | 0.0310 | -0.0119 | -0.0111 | -0.0103 |
| $x_{02}(10)$ | 0.6941 | 0.6941 | 0.7749 | 0.7831 | 0.7880 |
| $x_1(10)$ | 0.7812 | 0.7812 | 0.8256 | 0.8275 | 0.8281 |

TABLE 2: COMPARISON OF VARIOUS SERIES SOLUTIONS WITH THE EXACT SOLUTION FOR BVP

| x(k) | Degenerate Solution | Zero- order Solution | First- order Solution | Second-order Solution | Exact Solution |
|---|---|---|---|---|---|
| $x_{01}$ (0) | 20.0329 | 20.0329 | 19.8705 | 19.7962 | 19.6221 |
| $x_{02}(0)$ | -8.5442 | -8.5442 | -8.3187 | -8.0953 | -7.7110 |
| $x_1(0)$ | -8.0464 | 5.0000 | 5.0000 | 5.0000 | 5.0000 |
| | | | | | |
| $x_{01}$ (1) | 14.9988 | 14.9988 | 14.5555 | 14.6100 | 14.6338 |
| $x_{02}(1)$ | -4.3750 | -4.3750 | -3.4735 | -3.5742 | -3.6093 |
| $x_1(1)$ | -5.4219 | -5.4219 | -5.1814 | -5.1093 | -5.0501 |
| | | | | | |
| $x_{01}$ (2) | 11.2670 | 11.2670 | 11.0529 | 11.0965 | 11.1055 |
| $x_{02}(2)$ | -1.8011 | -1.8011 | -1.4098 | -1.4273 | -1.4820 |
| $x_1(2)$ | -2.3053 | -2.3053 | -2.3061 | -2.3155 | -2.3250 |
| | | | | | |
| $x_{01}$ (3) | 8.4823 | 8.4823 | 8.3943 | 8.4095 | 8.4347 |
| $x_{02}(3)$ | -0.2650 | -0.2650 | -0.1472 | -0.1636 | -0.1898 |
| $x_1(3)$ | -2.3053 | -2.3053 | -2.3061 | -2.3155 | -2.3250 |
| | | | | | |
| $x_{01}$ (4) | 6.3902 | 6.9302 | 6.3596 | 6.3905 | 6.4000 |

| | | | | | |
|---|---|---|---|---|---|
| $x_{02}(4)$ | 0.6042 | 0.6042 | 0.5871 | 0.5701 | 0.5621 |
| $x_1(4)$ | -1.4031 | -1.4031 | -1.4350 | -1.4399 | -1.4496 |
| | | | | | |
| $x_{01}$ (5) | 4.8082 | 4.8082 | 4.8086 | 4.8167 | 4.8421 |
| $x_{02}(5)$ | 1.0522 | 1.0522 | 0.9803 | 0.9751 | 0.9655 |
| $x_1(5)$ | -0.7665 | -0.7665 | -0.8044 | -0.3950 | -0.0178 |
| | | | | | |
| $x_{01}$ (6) | 3.6035 | 3.6.35 | 3.6147 | 3.6371 | 3.6437 |
| $x_{02}(6)$ | 1.2403 | 1.2403 | 1.1572 | 1.1509 | 1.1488 |
| $x_1(6)$ | -0.3156 | -0.3156 | -0.3495 | -0.3575 | -0.3602 |
| | | | | | |
| $x_{01}$ (7) | 2.6800 | 2.6800 | 2.6969 | 2.7051 | 2.7174 |
| $x_{02}(7)$ | 1.2733 | 1.2733 | 1.2019 | 1.1903 | 1.1973 |
| $x_1(7)$ | 0.0055 | 0.0055 | -0.0189 | -0.0259 | -0.0275 |
| | | | | | |
| $x_{01}$ (8) | 1.9694 | 1.9694 | 1.9896 | 1.9905 | 1.9985 |
| $x_{02}(8)$ | 1.2185 | 1.2185 | 1.1698 | 1.1670 | 1.1667 |
| $x_1(8)$ | 0.2357 | 0.2357 | 0.2213 | 0.2195 | 0.2155 |
| | | | | | |
| $x_{01}$ (9) | 1.4188 | 1.4188 | 1.4286 | 1.4315 | 1.4383 |
| $x_{02}(9)$ | 1.1187 | 1.1187 | 1.0949 | 1.0940 | 1.0933 |
| $x_1(9)$ | 0.4011 | 0.4011 | 0.3963 | 0.3951 | 0.3938 |
| | | | | | |
| $x_{01}$ (10) | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| $x_{02}(10)$ | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| $x_1(10)$ | 0.5210 | 0.5210 | 0.5274 | 0.5260 | 0.5252 |

## III. CONCLUSIONS

The dynamics of many continuous-time and discrete-time systems is described by high order differential equations. The solution of these high order stiff systems poses a problem and requires special numerical methods. SPM solves these problems by removing system's stiffness and reducing the order of the system. A SPM has been presented where the approximation solution has been obtained in terms of the outer solution and the one BLC solution corresponding to the small parameter. The BLC solution, obtained from the transformed systems, is meant mainly for the recovery of those initial and boundary conditions that are lost in the process of degeneration. SPM extended up to second-order approximation.

SPMs of discrete control systems are being developed but applications are not thoroughly explored. In order to fill this gap, here model of LFC of a single area power system is considered. The discrete model, obtained with a sampling period of 0.2s, possesses two-time-scale nature with two slow modes and one fast mode. It is modeled as a one parameter singularly perturbed system. Then an IVP & BVP are studied using the SPM for one parameter. The exact solution of BVP is obtained by the method of Complimentary Functions [21]. It is observed that it is very difficult to distinguish five solutions in one plot. Hence the results are shown in tables rather than graphs.

In most of the papers on SPM the results are shown up to first-order approximation only. Here we have presented the results up to second-order approximation giving more justification to the proposed method. As the order of approximation increases, the mean square error between the exact and approximate solutions decreases. It appears that the SPM is daunting due to the BLC equations. But it is not so as the corrections used in the SPM are to be calculated for only a limited number of values of k depending on the order of approximation. Also some functions of TSS may have trivial solutions as demanded by the selection of boundary conditions. Next we apply SPM to optimal control problems which are of practically important and difficult.

ACKNOWLEDGEMENTS

We greatly acknowledge Siddhartha Academy of General and Technical Education, Vijayawada for providing the facilities to carry out this research.

REFERENCES

Yuan Yuan, Fuchun Sun and Yenan Hu (2012), Decentralized multi-objective robust control of interconnected fuzzy singular perturbed model with multiple perturbation parameters. Fuzzy Systems (FUZZ-IEEE), IEEE International Conference, pp 1-8.

Xin, H., Gan, D. ,Huang, M. and Wang, K.(2010), Estimating the stability region of singular perturbation power systems with saturation nonlinearities: an linear matrix inequality based method. Control Theory & Applications, IET ,Vol. 4 , Issue 3, pp 351 – 361.

Baolin Zhang and MingQu Fan (2008), Near optimal control for singularly perturbed systems with small time-delay. Intelligent Control and Automation, WCICA 2008. 7th World Congress, pp 7212 – 7216

Dmitriev M. and Kurina G. (2006), Singular perturbations in control problems. *Automation and Remote Control*, 67, 1, 1-43.

Yong Chen and Yongqiang Liu (2005), Summary of Singular Perturbation Modeling of Multi-time Scale Power Systems. Transmission and Distribution Conference and Exhibition: Asia and Pacific, IEEE/PES, pp 1-4.

Naidu D. S. (2002), Singular Perturbations and Time Scales in Control Theory and Applications: An Overview. Dynamics of Continuous, Discrete & Impulsive Systems, 9, 2, 233-278.

Kevorkian J. K. and Cole J. D.(1996). Multiple Scale and Singular Perturbation Methods. Springer-Verlag, New York.

Saksena V. R., O'Reiley J. and Kokotovic P. V. (1984), Singular Perturbations and Time-Scale Methods in Control Theory: Survey 1976-1983. Automatica, 20, 3, 273-293.

Naidu, D.S and Rao, A.K. (1985), Singular perturbation analysis of discrete control systems. Volume 1154 of Lecture Nots in Mathematics, A.Dold and B. Eckmann, eds, Springer-Verlag

Naidu, D.S and D.B Price (1988), Singular perturbations and time scales in the design of digital flight control systems. NASA Technical paper 2844.

Krishnarayalu M. S. and Naidu D. S. (1987), Singular perturbation method for initial value problems in two-parameter discrete control systems. Int. J. Systems Science, 18, 12, 2197-2208.

Krishnarayalu M. S. and Naidu D. S. (1988), Singular perturbation method for boundary value problems in two-parameter discrete control systems. Int. J. Systems Science, 19, 10, 2131-2143.

Krishnarayalu M. S. (1989), Singular perturbation method applied to the open-loop discrete optimal control problem with two small parameters. Int. J. Systems Science, 20, 5, 793-809.

Krishnarayalu M. S. (1990), Singular perturbation method applied to the closed-loop discrete optimal control problem. Optimal Control Applications & Methods, 11, 1, 75-83.

Krishnarayalu M. S. (1994), Singular perturbation analysis of a class of initial and boundary value problems in multiparameter digital control systems. Control-Theory and Advanced Technology, 10, 3, 465-477.

Krishnarayalu M. S. (1999), Singular perturbation methods for one-point, two-point and multi-point boundary value problems in multiparameter digital control systems. Journal of Electrical and Electronics Engineering, Australia, 19, 3, 97-110.

Krishnarayalu M. S. (2004), Singular perturbation methods for a class of initial and boundary value problems in multi-parameter classical digital control systems. ANZIAM J., 46, 67-77.

Krishnarayalu M. S. (2008), Singular perturbation method applied to the discrete Euler-Lagrange free-endpoint optimal control problem. Automatic Control (theory and applications) AMSE journal, 63, 3, 16-29.

Kishore Babu G. and Krishnarayalu M. S. (2012), An Application Of Discrete Two Parameter Singular Perturbation Method, IJERT, Vol. 1 Issue 10.

B.Venkata Prasanth (2008), Load frequency control for a two area power system using robust genetic algorithm controller. JATIT, Vol. 4, No. 12.

Roberts S.M. and Shipman J.S. (1972) Two-point Boundary Value Problems: Shooting Methods. Elsevier, New York.

Koichi F. and Kunihiko K. (2003). Bifurcation cascade as chaotic itinerancy with multiple time scales. Chaos: An Interdisciplinary Journal of Nonlinear Science, 13, 1041-1056.


**Mr. Kishore Babu G.** is pursuing his Ph.D. from Nagarjuna University, AP, India. He is also working as an Associate Professor in the department of EEE at PSCMRCET, Vijayawada, India. He was born in India on August 20 1986. He obtained his graduation in EEE in April 2006 and post graduation in PE & ED from JNTU-Hyderabad in August 2008. His research interests are Control Systems, Power Systems and Power Electronic Drives.

Dr. M.S. Krishnarayalu was born in 1955, in Andhra Pradesh, India. He received B.E., M. Tech. and Ph. D. degrees in Electrical Engineering in 1977, 1979 and 1990 from Andhra University, Vishakhapatnam, Jawaharlal Nehru Technological University, Anantapur and Indian Institute of Technology, Kharagpur respectively. Currently he is working as professor in Electrical and Electronics Engineering Department of V. R. Siddhartha Engineering College. His research areas of interest are Power Systems and Control Systems. He is a member of ResearchGate with RG score of 7.32.