

Stochastic Modeling for The Daily Extreme Flows of Kelkit Stream

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Abstract: The aim of this study is to determine whether the daily extreme flows for Kelkit Stream could be forecast as a stochastic model. For this aim, the autoregressive models (the first and second order Markov models) and Arima(1.0.1) model (mixed autoregressive-moving average model) were used. The flows forecasted by using the models mentioned were compared to the observed flows. The results showed that the flow predictions based on the first order Markov model are fitted to the data better than the other models.

Key words: Extreme flows, autocorrelation coefficient, Markov model, Arima model.

Kelkit Çayının Günlük Ekstrem Akımları için Stokastik Modelleme

Özet: Bu çalışmanın amacı, Kelkit çayı günlük ekstrem akımlarının stokastik bir modelle tahmin edilebilirliğini belirlemek içindir. Bu amaçla otoregresif modeller (birinci ve ikinci derece Markov modelleri) ve Arima(1.1) model (otoregresif-hareketli ortalama) kullanılmıştır. Adı geçen modeller kullanılarak tahmin edilen akımların gözlenen akımlar ile karşılaştırılması yapılmıştır. Sonuçlar, birinci derece Markov modelden elde edilen akım tahminlerinin, veriye diğer modellerden daha fazla uyum sağladığını göstermiştir.

Anahtar kelimeler: Ekstrem akımlar, otokorelasyon katsayısı, Markov model, Arima model.

Introduction

Hydrologic phenomena are cyclic and stochastic in nature. The rotation of the earth around the sun is one of the main factors that produces cyclicities; meanwhile, erratic atmospheric movement contribute to the randomness of natural hydrologic processes. Therefore, both the cyclicity and the stochasticity in the hydrologic processes are equally important in the modeling studies (1).

A time series may consist only deterministic events, only stochastic events or a combination of the two. Most often a hydrologic time series will be composed of a stochastic component superimposed on a deterministic component. Ultimately design decisions must be based on a stochastic model or a combination of stochastic and deterministic models. This is because any system must be designed to operate in the future. A stochastic model is a probabilistic model having parameters that must be obtained from observed data. Stochastic stream flows are neither historical flows nor predictions of future flows, but they are representative of possible future flows in a statistical sense (2).

Most of the statistical methods used in hydrologic studies are based on the assumption that the observations are independently distributed in time. The occurrence of an event is assumed to be independent of all previous events. This assumption is not always valid for hydrologic time series. Observations of daily discharges do not change appreciably from one day to the next. There is a tendency for the values to cluster, in the sense that high values tend to follow high values and low values tend to follow low values. Thus the daily discharges are not independently distributed in time. The dependence among monthly discharges is less than that between daily discharges, and the dependence among annual discharges is less than that between monthly discharges. Thus the dependence between hydrologic observations decreases with an increase in the time base (3).

If there is a linearly dependence between the values of a time series, these series are called as a

stochastic process or autoregressive process. The correlation between the values may be taken as a dependence criterion.

It is inadequate to determine only the probability distribution fitted to the stochastic process in hydrologic studies, meanwhile, the stochastic process must be modeled according to the dependence between the values of the time series. The modeling stochastic process is for generating data. In general, the period for which data is available is usually less than the economic life of the project in the studies involving planning and management of water resources. However, McMichael and Hunter (4) stated that providing good forecast functions for time dependent data was a common problem.

Material and Methods

In this study, the daily flow values measured from 1938 to 1988 in the flow station numbered 1401 controlled by General Directorate of Electric Power Research Survey and Development Administration (EIE) were used. The flow station was nearby Fatli Bridge in Tokat. Kelkit stream is formed by joining together of small streams that originate from Spikor, Pular, Otlukbeli, Sarhan and Balaban mountains, located in the north Erzincan, near the Kelkit district. Kelkit stream passes through Susehri, Niksar and Erbaa plains and then, joins to Yesilirmak River in the north of Erbaa plain. Kelkit stream is 245.5 km in length and its watershed area is 11455 km² (5).

Determination of Daily Extreme Flows

The daily flow data were used to model the daily extreme flows of Kelkit stream as a stochastic process. Although the flow records are available in the mentioned flow station, the measurements after 1988 were discarded since Kilickaya dam was built on Kelkit stream in that year. Okman (6) stated that the flows were homogenous from 1938 to 1988.

The daily extreme flows for each year was taken from the daily flows recorded from 1938 to 1988, and the remaining data were disregarded (7). Thus, the daily extreme flows taken were supposed to be a random and continuous variable that can represent the discarded daily flows (8).

Modeling Stochastic Process of The Daily Extreme Flows

The First and Second Order Markov Models

Most hydrologic time series exhibit significant serial correlation. That is, the value of the random variable under consideration at one period is correlated with the values of the random variable at earlier time periods. The correlation of a random variable X at one time period with its value k time periods earlier is denoted by r_k and is called the k^{th} order serial correlation (9).

A first order Markov model is defined by the equation (10)

$$Z_{i+1} = \mu_z + (r_1)(Z_i - \mu_z) + \varepsilon_{i+1} \quad (1)$$

A second order Markov model is defined by the equation (11).

$$Z_{i+2} = \mu_z + \beta_1(Z_{i+1} - \mu_z) + \beta_2(Z_i - \mu_z) + \varepsilon_{i+2} \quad (2)$$

β_1 and β_2 are,

$$\beta_1 = (r_1 - r_1 r_2) / (1 - r_1^2) \quad (3)$$

$$\beta_2 = (r_2 - r_1^2) / (1 - r_1^2) \quad (4)$$

r_k in equations is referred to as k^{th} order serial correlation coefficient (autocorrelation coefficient). k^{th} order serial correlation coefficient may be calculated according to circular series or open series approach (11).

A circular series is one that closes on itself so that z_n is followed by z_1 (4). k^{th} order serial correlation coefficient based on circular series approach is

$$r_k = \frac{\sum_{i=1}^n z_i z_{i+k} - n \mu_z^2}{(n-1) s_z^2} \quad (5)$$

Matalas (12) has suggested that for hydrologic data r_k tends to be greater than zero due to persistence caused by storage. If the r_k is zero, there is a linearly independence between the values of a time series. There is a high dependence between the values of a time series if the r_k is between zero and minus or plus 1 (3).

Equation 1 and 2 give suitable results when the data is normally distributed. Therefore, Hipel et al. (13) suggested the use of transformed data in autoregressive models since most hydrologic events have a skewed distribution. These transformations are as follows;

$$z_{i=1}^n = \lambda^{-1} \left[(x_{i=1}^n + c) - 1 \right] \quad \lambda \neq 0 \quad (6)$$

$$z_{i=1}^n = \ln(x_{i=1}^n + c) \quad \lambda = 0 \quad (7)$$

McLeod et al. (14) stated that λ might be ± 5 or ± 1 , and C might be 1.

In Haan (2), with respect to the first order serial correlation coefficient r_1 , Anderson showed that, since r_1 is nearly normally distributed, the confidence limits (CL) for a computed value of r_1 are given by

$$CL = \left(-1 \pm t_{1-\alpha/2} \sqrt{(n-2)} \right) / (n-1) \quad (8)$$

For the confidence limits of the serial correlation coefficients higher than r_1 , $n-k+1$ must be substituted for n in equation 8 (15; 16).

Bayazit (15) stated that the second order Markov model is preferred to the first order Markov model when $R^2 - r_1^2$ is higher than $0.01r_1^2$. R^2 is equal to $(r_1^2 + r_2^2 - 2r_1^2 r_2) / (1 - r_1^2)$.

The Arima Model

Arima model is an integration of autoregressive models and moving average models (17). Arima model is expressed as;

$$Z_i = \phi_1 Z_{i-1} + \dots + \phi_p Z_{i-p} + \varepsilon_i - \theta_1 \varepsilon_{i-1} - \dots - \theta_q \varepsilon_{i-q} \quad (9)$$

The model has $p+q+2$ parameters. This model is also called as Arima (p.d.q). The Arima (1.0.1) is extensively used for the sequences of annual discharge volumes (15).

Carlson (18) showed that Arima(1.0.1) could be converted to the autoregressive model. Thus, if a value for θ_1 is assumed, say, $\theta_1 = \theta^0$, then the data $z_1, z_2, z_3, \dots, z_n$ can be converted to a new data set $w_1, w_2, w_3, \dots, w_n$ according to

$$W_n = Z_n + \theta^0 W_{n-1} \quad (10)$$

The new set of data w_n , now follows the pure autoregressive model

$$W_i = \phi W_{i-1} + \varepsilon_i \quad (11)$$

Tao and Delleur (1) obtained ϕ_1, θ_1 parameters for Arima(1.0.1) as follows

$$r_1 = \frac{(1 - \phi_1 \theta_1)(\phi_1 - \theta_1)}{1 + \phi_1^2 - 2\phi_1 \theta_1} \quad (12)$$

$$r_k = \phi_1 r_{k-1} \quad k \geq 2 \quad (13)$$

Generation of The Residuals (ε_i)

ε_i is an independently distributed random variable. Average of this variable is zero and its variance is σ_ε^2 . It is assumed that ε_i is normally distributed (19).

The numbers of ε_i can be obtained as follows (2)

$$\varepsilon_i = \sigma_\varepsilon R_n + \mu_\varepsilon \quad (14)$$

R_n is a random standard normal deviate and is a random observation from a standard normal distribution. There is a table prepared for random standard normal deviates.

The variances of ε_i for the first and second order Markov models can be calculated from the following relationships, respectively (20; 11).

$$\sigma_\varepsilon^2 = \left(\frac{n-1}{n}\right)(1-r_1^2) \sum_{i=1}^n (z_i - \mu_z)^2 \frac{1}{(n-3)} \tag{15}$$

$$\sigma_\varepsilon^2 = \frac{(n-2)}{(n-3)} [c_0 - \beta_1 c_1 - \beta_2 c_2] \tag{16}$$

$$r_1 = \left(\frac{c_1}{c_0}\right) = \frac{\sum_{i=1}^{n-1} (z_i - \mu_z)(z_{i+1} - \mu_z)}{\left(\sum_{i=1}^n (z_i - \mu_z)^2\right)} \tag{17}$$

$$r_2 = \left(\frac{c_2}{c_0}\right) = \frac{\sum_{i=1}^{n-2} (z_i - \mu_z)(z_{i+2} - \mu_z)}{\left(\sum_{i=1}^n (z_i - \mu_z)^2\right)} \tag{18}$$

The variance (σ_ε^2) for the Arima (1,1) model can be calculated from the following equation (9),

$$\sigma_w^2 = \frac{\sigma_\varepsilon^2}{1 - \phi_1^2} \tag{19}$$

Results and Discussion

Three models have been used for forecasting of the daily extreme flows of Kelkit stream. In forecasting the daily extreme flows for Kelkit stream, serial correlation coefficient was taken as a base for each model.

The daily extreme flows were transformed to be fitted normal distribution, by taking λ equal to $\pm 5, \pm 1$ or zero in Equation 6 and 7. The calculated flows for $\lambda, \pm 5, \pm 1$ and zero were tested by Simirnov-Kolmogorov method. The calculated flows for +5 were more normally distributed than the others. Simirnov-Kolmogorov test results are given in Table 1.

The serial correlation coefficients were calculated from the Equation 5 for the transformed daily extreme flows. These serial correlation coefficients are given in Table 2. As it can be seen in Table 2, the serial correlation coefficients for the daily extreme flows were between -0.2539 and 0.2131 for the maximum daily flows, were between -0.3085 and 0.5659 for the minimum daily flows. The correlograms for the daily extreme flows were illustrated in Figure 1. Although the serial correlation coefficients for the daily extreme flows are low (close to zero) at some years, it can be said to be dependence among the observations.

To test whether the series of the daily extreme flows are dependent or independent, CL values for 5 % confidence limits were calculated using Equation 8. These values were shown in Table 3. Box and Jenkins (9) expressed that if 95 % of the calculated twenty serial correlation coefficients are within the confidence limits, then the series of the daily extreme flows will be independent. In this study, all of the calculated serial correlation coefficients for the daily maximum flows were within the confidence limits. But less than 95 % of the calculated serial correlation coefficients for the daily minimum flows were within the confidence limits. Therefore, it can be said that the daily maximum flows are independent and the daily minimum flows are dependent. But, as it can be seen the serial correlation coefficient of the daily maximum flows, there is dependence among the daily maximum flows. As a result of these, the autoregressive models for the daily extreme flows were used.

Table 1. Simirnov-Kolmogorov Test Results from Normal Distribution for λ .

λ	Maximum Flow		Minimum Flow		$\Delta = \max_i F(x_i) - F_a(x_i) $	
	F(x _i)	F _a (x _i)	F(x _i)	F _a (x _i)	Maximum Flow	Minimum Flow
0	0.89285	0.96078	0.73005	0.66667	0.0679	0.0634
0.5	0.90731	0.96078	0.37633	0.43137	0.0535	0.0550
-0.5	0.53785	0.45098	0.55117	0.47059	0.0869	0.0806
1.0	0.64999	0.70588	0.35400	0.43137	0.0559	0.0774
-1.0	0.70264	0.96078	0.57738	0.47059	0.2581	0.1068

F(x_i) is the probability levels from normal distribution for the observations.
F_a(x_i) is the frequencies based on the ranks of the observations

Table 2. Serial Correlation Coefficients for Kelkit Stream's Daily Extreme Flows

r_k	Q_{max} Q_{min}		r_k for Markov Models				Arima Model
			r_{emax1}	r_{emax2}	r_{emin1}	r_{emin2}	r_k for Q_{min}
r_1	0.0583	0.5659	-0.0363	0.0474	-0.0492	-0.0591	-0.0028
r_2	0.1986	0.3679	0.1933	-0.0147	-0.0897	-0.1324	-0.1127
r_3	-0.1849	0.3779	-0.1974	-0.1628	0.2020	0.1620	0.1771
r_4	0.2131	0.2849	0.2434	0.1521	0.1023	0.0356	0.0657
r_5	-0.1663	0.1297	-0.1968	-0.1539	-0.1550	-0.1106	-0.1508
r_6	0.1193	0.1450	0.1141	0.0709	0.1187	0.0730	0.0847
r_7	-0.1743	0.1326	-0.2028	-0.1330	-0.0141	-0.0194	-0.0206
r_8	0.1208	0.1433	0.1803	0.0721	0.0581	0.0854	0.0510
r_9	-0.1451	0.1288	-0.2215	-0.1495	0.1275	0.1562	0.1214
r_{10}	-0.0130	0.0238	0.0805	0.0375	-0.0440	-0.1127	-0.0639
r_{11}	-0.1141	0.0029	-0.1798	0.0183	-0.1401	-0.0562	-0.1217
r_{12}	0.0510	0.0854	0.2140	0.0897	0.2528	0.2192	0.2458
r_{13}	-0.1955	0.0002	-0.3218	-0.2337	-0.0645	-0.0724	-0.0432
r_{14}	-0.0321	-0.0559	0.0466	-0.0563	-0.0530	-0.0447	-0.0401
r_{15}	-0.0312	-0.0829	-0.1027	0.0885	0.0637	0.0559	0.0567
r_{16}	-0.0510	-0.2127	0.1010	-0.1106	-0.1344	-0.2076	-0.1692
r_{17}	-0.2539	-0.2690	-0.3795	-0.3462	-0.2104	-0.1012	-0.1919
r_{18}	0.0159	-0.2522	0.1549	0.0662	0.0998	0.1119	0.1155
r_{19}	-0.0875	-0.3085	-0.1518	0.1379	-0.1422	-0.1095	-0.1286
r_{20}	0.0601	-0.3064	0.2339	0.0791	-0.0370	-0.1530	-0.0388

Table 3. The Statistics for The Models in the Study

Parameter	Maximum Flows		Minimum Flows		Arima Model			
	MM_I	MM_{II}	MM_I	MM_{II}	Maximum Flows		Minimum Flows	
B_1	***	0.0469	***	0.5262	***		***	
B_2	***	0.1959	***	0.0701	***		***	
CL_F	+0.25	-0.29	+0.25	-0.29	+0.25	-0.29	+0.25	-0.29
CL_E	+0.26	-0.30	+0.26	-0.30	***		+0.26	-0.30
Σ^2	39.37	1974.2	0.58	28.24	-482.6		0.56	
θ_1	***	***	***	***	3.407		0.650	
θ_2	***	***	***	***	0.299		0.124	

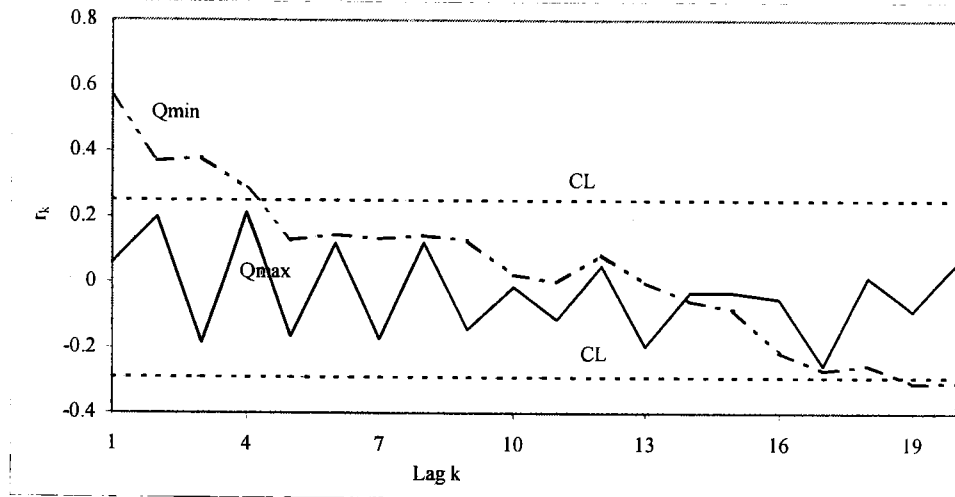


Figure 1. The Correlogram for The Daily Maximum and Minimum Flows

To determine whether which one of the first order and second order Markov models are suitable, the residuals (ϵ_i) for the daily extreme flows were calculated using Equation 1 and 2. The serial correlation coefficients of the ϵ_i were calculated Equation 5. These serial correlation coefficients are given in Table 2. The correlograms of the ϵ_i are illustrated in Figure 2 and Figure 3. To test whether the ϵ_i is dependent or independent, CL values for 5 % confidence limits were calculated using Equation 8. These values were shown in Figure 2 and Figure 3. As it can be seen in Figure 2, less than 95 % of the serial correlation coefficients of the ϵ_i calculated from the first order Markov model for the daily maximum flows were within the confidence limits. But, 95 % of the serial correlation coefficients of the ϵ_i calculated from the second order Markov model for the daily maximum flows were within the confidence limits (Figure 3). Therefore, for forecasting of the daily maximum flows, it can be expressed that the first order Markov model is not suitable, but the second order Markov model is suitable. As it can be seen in Figure 2 and Figure 3, all of the serial correlation coefficients of the calculated ϵ_i from the first and second order Markov models for the daily minimum flows were within the confidence limits. Thus, the first and second order Markov models can be used for forecasting the daily minimum flows.

To determine which one of the first or second order Markov models is suitable for the series of the daily extreme flows, it is necessary to determine what the relationship between $R^2 - r_1^2$ and $0.01 r_1^2$ is. $R^2 - r_1^2$ was higher than $0.01 r_1^2$ for the daily extreme flows. Therefore, the second order Markov model was preferred to the first order Markov model, because the second order Markov model reflects the variance of the variable (the daily extreme flows) better than the first order Markov model.

The first and second order Markov model equations for the daily maximum and minimum flows of Kelkit stream are as follows, respectively.

For the first order Markov model,

$$Z_{i+1} = 41.96 + (0.0583)(Z_i - 41.96) + \epsilon_{i+1} \quad (20)$$

$$Z_{i+2} = 4.52 + (0.5659)(Z_i - 4.52) + \epsilon_{i+1} \quad (21)$$

For the second order Markov model,

$$Z_{i+2} = 41.96 + 0.0469(Z_{i+1} - 41.96) + 0.1959(Z_i - 41.96) + \epsilon_{i+2} \quad (22)$$

$$Z_{i+2} = 4.52 + 0.5262(Z_{i+1} - 4.52) + 0.0701(Z_i - 4.52) + \epsilon_{i+2} \quad (23)$$

Although the second order Markov model is mathematically more suitable than the first order Markov model ($R^2 - r_1^2 > 0.01 r_1^2$), the daily extreme flows from the first order Markov model reflects the observed flows well than the second order Markov model (Figure 4, Figure 5, Figure 6, Figure 7). Bayazit (20), Janacek and Swift (10) stated that while number of parameters in a model increases, the success of prediction decreases.

Arima(1.0.1) model is not suitable for forecasting the daily maximum flow of Kelkit stream. ϕ_1 parameter was obtained as 3.407 (Table 3). That is, this parameter was higher than 1. Box and Jenkins (9) reported that the absolute values of ϕ_1 and θ_1 parameters must be between zero and one. Because of the ϕ_1 , the variance of the ϵ_i was negative (Table 3). But, forecasting the daily minimum flows of Kelkit stream are suitable with Arima(1.0.1) model. ϕ_1 and θ_1 parameters were lower than 1 (Table 3). But, as it can be seen in Figure 8, the calculated flows were smaller than the observed flows. All of the correlation coefficients of the ϵ_i calculated based on Arima(1.0.1) for the daily minimum flows were within the confidence limits (Table2).

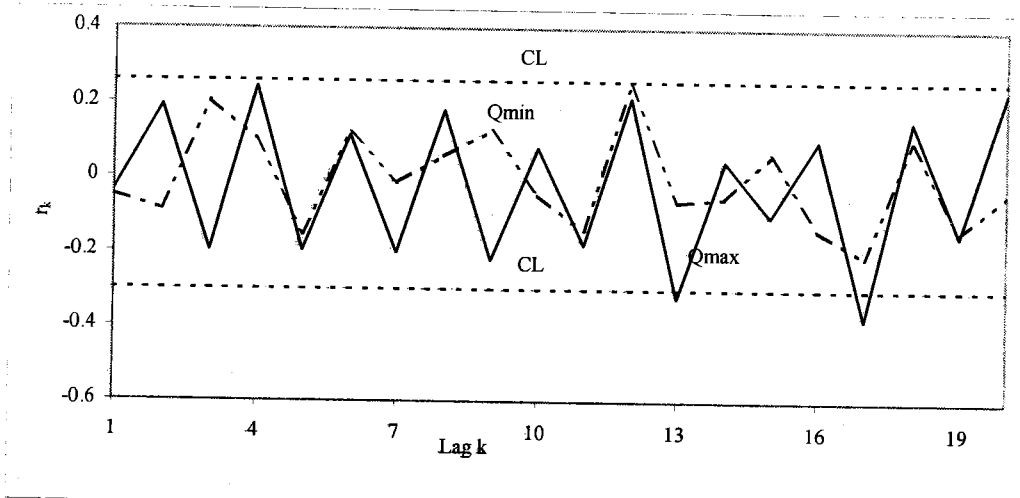


Figure 2. The Correlogram of The Residuals of The Daily Extrem Flows for The First Order Markov Model

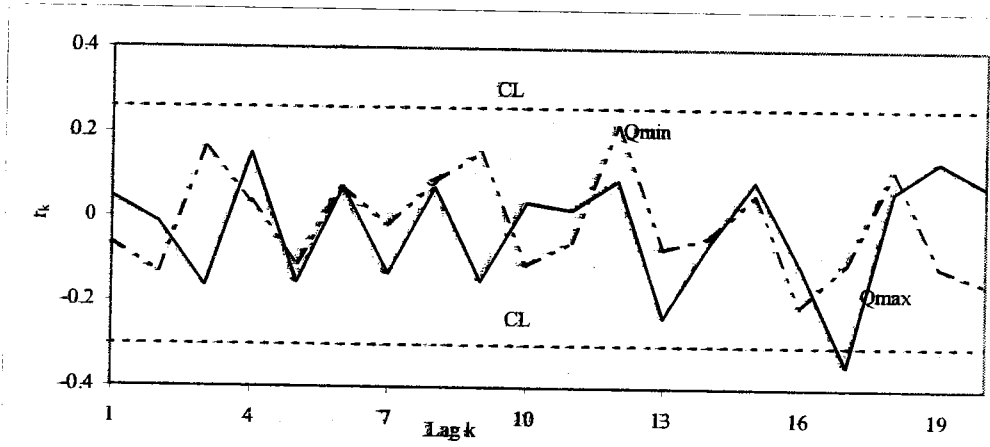


Figure 3. The Correlogram of The Residuals of The Daily Extrem Flows for The Second Order Markov Model

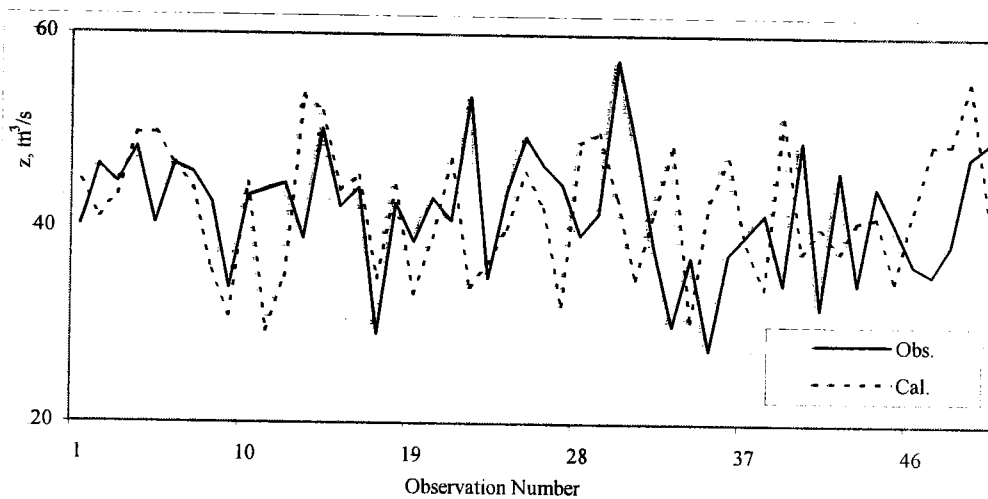


Figure 4. The Curve of The Daily Maximum Flows for The First Order Markov Model

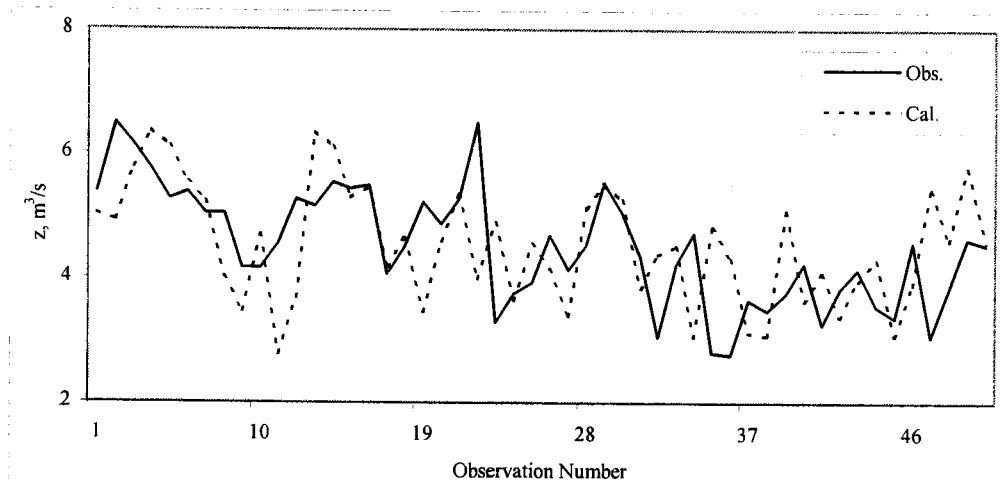


Figure 5. The Curve of The Daily Minimum Flows for The First Order Markov Model

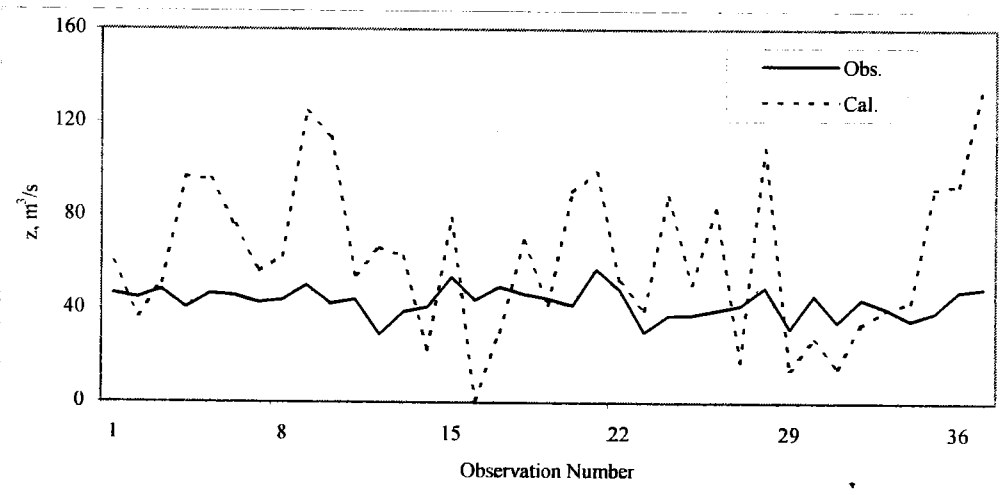


Figure 6. The Curve of The Daily Maximum Flows for The Second Order Markov Model

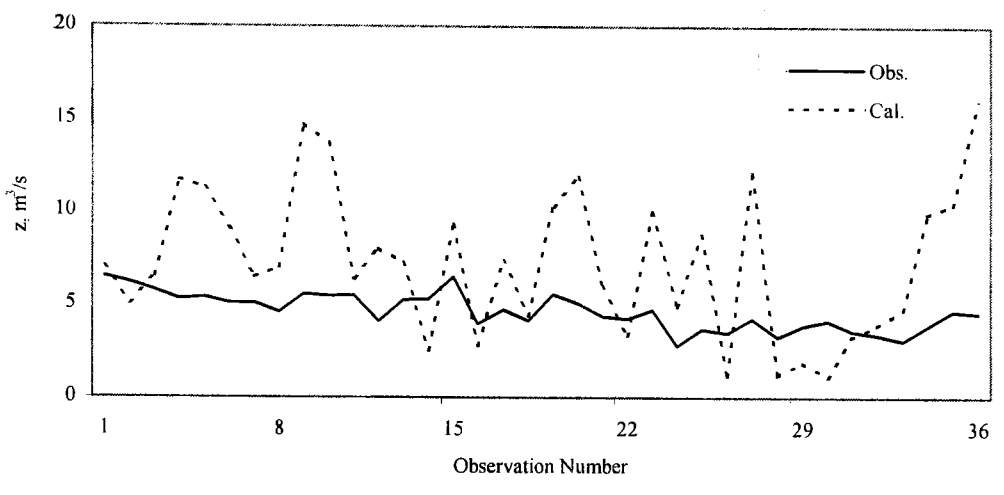


Figure 7. The Curve of The Daily Minimum Flows for The Second Order Markov Model

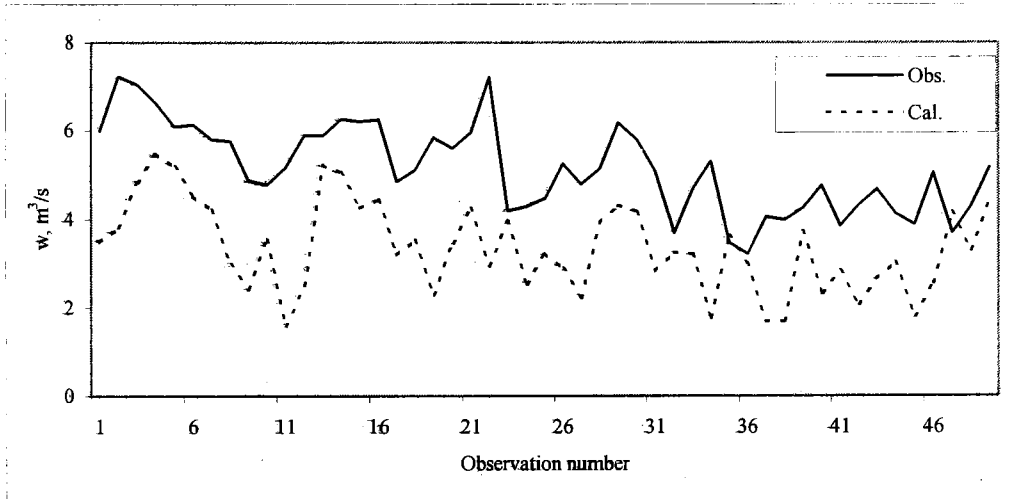


Figure 8. The Curve of The Daily Minimum Flows for The Arima Model

Notation

The following symbols are used in this paper.

x_i	= The observed flows
z_i	= The transformed series by λ
μ_z	= The average of z_i
r_k	= k th-order serial correlation coefficient
ε_i	= Normally independently distributed random variable
β_1, β_2	= Parameters for the second order Markov model
n	= Sample size
s_z	= The standard deviation of z_i
CL	= Confidence limit for 5 %
$t_{1-\alpha/2}$	= The t value
ϕ_1, θ_1	= Parameters for the Arima(1.0.1) model
w_n	= The converted series by θ_1
σ_ε^2	= The variance of ε_i
μ_ε	= The average of ε_i
R_n	= Standard normal deviate
c_0, c_1, c_2	= Parameters for the second order Markov model
σ_w^2	= The variance of w_n
$r_{\text{emax}1}$	= The r_k of the daily maximum flow residuals for MM_I
$r_{\text{emax}2}$	= The r_k of the daily maximum flow residuals for MM_{II}
$r_{\text{emin}1}$	= The r_k of the daily minimum flow residuals for MM_I
$r_{\text{emin}2}$	= The r_k of the daily minimum flow residuals for MM_{II}
CL _F	= Confidence limit for the flows
CL _{ε}	= Confidence limit for the residuals
MM_I	= The first order Markov model
MM_{II}	= The second order Markov model

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