İstanbul Üniv. Fen. Fak. Mat. Der. **60** (2001), 101-109

ON THE UPPER LOWER SUPER D-CONTINUOUS MULTIFUNCTIONS

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ABSTRACT. In this paper, we define upper (lower) Super D- continuous multifunctions and obtain some characterizations and some basic properties of such a multifunction. Also some relationships between the concept of Super D-continuity and known concepts of continuity and weak continuity are given.

1. INTRODUCTION

In 1968, Singal and Singal [9] introduced and investigated the concept of almost continuous functions. In 1981, Helderman [2] introduced some new regularity axioms and studied the class of D-regular spaces. In 2001, J. K. Kohli [3] introduced the concept of D-supercontinuous functions and some properties of D-supercontinuous functions are given by him.The purpose of this paper is to extend this concept and to give some results for multifunctions.

A multifunction $F : X \rightsquigarrow Y$. is a correspondence from X to Y with $F(x)$ a nonempty subset of Y, for each $x \in X$. Let A be a subset of a topological space (X, τ). $\overset{o}{A}$ and $\overline{A}$ (or $intA$ and clA) denote the interior and closure of A respectively. A subset A of X is called regular open (regular closed) [10] iff $A = int(cl(A))$ $(A = cl(int(A)))$. A space (X, τ) is said to be almost regular [8] if for every regular closed set F and each point x not belonging to F, there exist disjoint open sets U and V containing F and x respectively. For a given topological space (X, τ), the collection all sets of the form $U^+ = \{T \subseteq X : T \subseteq U\}$ $(U^- = \{T \subseteq X : T \cap U \neq \emptyset\})$ with U in τ, form a basis (subbasis) for a topology on 2^X, where 2^X is the set of all nonempty subset of X (see [5]). This topology is called upper (lower) Vietoris topology and denoted by τ_V^+ (τ_V^-). A multifunction F of a set X into Y is a correspondence such that $F(x)$ is a nonempty subset of Y for each $x \in X$. We will denote such a multifuntion by $F : X \rightsquigarrow Y$. For

1991 *Mathematics Subject Classification.* Primary 54C10; Secondary 54C60.
Key words and phrases. multifunction, D- continuity, continuity,Super D-Continuity.

a multifunction F, the upper and lower inverse set of a set B of Y will be denoted by $F^+(B)$ and $F^-(B)$ respectively that is $F^+(B) = \{x \in X : F(x) \subseteq B\}$ and $F^-(B) = \{x \in X : F(x) \cap B \neq \emptyset\}$. The graph $G(F)$ of the multifunction $F : X \longrightarrow Y$ is strongly closed [4] if for each $(x, y) \notin G(F)$, there exist open sets U and V containing x and containing y respectively such that $(U \times \overline{V}) \cap G(F) = \emptyset$.

Definition A. [7] A multifunction $F : X \rightsquigarrow Y$ is said to be

(a) upper semi continuous (briefly u.s.c.) at a point $x \in X$ if for each open set V in Y with $F(x) \subseteq V$, there exists an open set U containing x such that $F(U) \subseteq V$;

(b) lower semi continuous (briefly l.s.c.) at a point $x \in X$ if for each open set V in Y with $F(x) \cap V \neq \emptyset$, there exists an open set U containing x such that $F(z) \cap V \neq \emptyset$ for every $z \in U$.

2. Super D-Continuous Multifunctions

Definition 1. *Let X and Y be two topological spaces.*

a)A multifunction $F : X \rightsquigarrow Y$ is said to be upper D-super continuous (u.D-sup.c.) at a point $x_0 \in X$ if for every open set V with $F(x_0) \subset V$,there exists an open F_σ-set U_{x_0} containing x_0 such that the implication $x \in U_{x_0} \Rightarrow F(x) \subset V$ is holds.

b)A multifunction $F : X \rightsquigarrow Y$ is said to be lower D-super continuous(l.D-sup.c.) at a point $x_0 \in X$ if for every open set V with $F(x_0) \cap V \neq \emptyset$, there exists an open F_σ-set U_{x_0} containing x_0 such that the implication $x \in U_{x_0} \Rightarrow F(x_0) \cap V \neq \emptyset$ is holds.

c)A multifunction $F : X \rightsquigarrow Y$ is upper D-super continuous(resp.lower D-super continuous) if it has this property at each point $x \in X$.

Definition 2. *A set G in a topological space X said to be d-open if for each $x \in G$, there exists an open F_σ-set H such that $x \in H$ and $H \subseteq G$. The complement of a d-open set will be referred to as a d-closed set. [Kohli, D-supercontinuous Functions]*

Theorem 1. *For a multifunction $F : X \rightsquigarrow Y$, the following statements are equivalent.*

a)F is u.D.sup.c.(l.D.sup.c.)

b)For each open set $V \subseteq Y, F^+(V)$ $(F^-(V))$ is a d-open set in X.

c)For each closed set $K \subseteq Y, F^-(K)$ $(F^+(K))$ is a d-closed set in X.

d)For each x of X and for each open set V with $F(x) \subset V (F(x) \cap V \neq \emptyset)$,there is a d-open F_σ-set U containing x such that the implication $y \in U \Rightarrow F(y) \subset V$ is holds $(F(y) \cap V \neq \emptyset)$.

We omit the proof.

Following example gives that u.s.c.(l.s.c.) does not imply u.$D-$sup.c.(l.D $-$sup.c.)

Example 1. $X = \{0,1\}$, $\tau = \{\emptyset, X, \{0\}\}$, $Y = \{a,b,c\}$, $\gamma = \{\emptyset, Y, \{a\}, \{a,b\}\}$. $F(0) = \{a\}$, $F(1) = \{b\}$. *F is u.s.c.(l.s.c.) but not u.D.sup.c.(l.D. sup.c.). $F^+(\{a\}) = \{0\}$ is open but not F_σ-open $F^-(\{a\}) = \{0\}$*

Definition 3 (2). *A space X is called D-regular if X has a base consisting of open F_σ-sets.*

Theorem 2. *Every u.s.c. multifunction on a D-reguler space is u.D.sup.c.*

Proof. Since every open set is d-open in a D-regular space, the proof is clear. □

Definition 4. *Let X be a topological space and let $A \subset X$. A point $x \in X$ is said to be a d-adherent point of A if every open F_σ-set containing x intersects A. Let $[A]_d$ denote the set of all d-adherent points of A. Clearly the set A is d-closed if and only if $[A]_d = A$.[Kohli, D-supercontinuous Functions]*

Theorem 3. *A multifunction $F : X \rightsquigarrow Y$ is l.D.sup.c. if and only if $F([A]_d) \subset \overline{F(A)}$ for every $A \subset X$.*

Proof. Suppose F is l.D.sup.c.. Since $\overline{F(A)}$ is closed in Y by Theorem(1) $F^+(\overline{F(A)})$ is d-closed in X. Also since $A \subset F^+(\overline{F(A)})$, $[A]_d \subset [F^+(\overline{F(A)})]_d$ $= F^+F([A]_d)$ Thus $F([A]_d) \subset F(F^+(\overline{F(A)})) \subset \overline{F(A)}$.

Conversely, suppose $F([A]_d) \subset \overline{F(A)}$ for every $A \subset X$. Let K be any closed set in Y.Then $F([F^+(K)]_d) \subset \overline{F(F^+(K))}$ and $\overline{F(F^+(K))} \subset \overline{K} = K$.Hence $[F^+(K)]_d \subset F^+(K)$ which shows that F is l.D. sup .c. □

Theorem 4. *A multifunction F from a space X into a space Y is l.D.sup.c. if and only if $[F(B)]_d \subset F(\overline{B})$ for every $B \subset Y$.*

Proof. Suppose F is l.D.sup.c.. Then $F^+(\overline{B})$ is d-closed in X for every $B \subset Y$ and $F^+(\overline{B}) = [F^+(\overline{B})]_d$. Hence $[F^+(B)]_d \subset F^+(\overline{B})$

.Conversely, let K be any closed set in Y. Then $[F^+(K)]_d \subset F^+(\overline{K}) = F^+(K) \subset [F^+(K)]_d$. Thus $F^+(K) = [F^+(K)]_d$ which in turn implies that F is l.D.sup.c. □

Definition 5. *A filter base $\mathcal{F}$is said to d-converge to a point x (written as $\mathcal{F} \xrightarrow{d} x$) if for every open F_σ-set containing x contains a member of T $-$.[Kohli, D-supercontinuous Functions].*

Theorem 5. *A multifunction $F : X \rightsquigarrow Y$ is l.D.sup.c. if and only if for each $x \in X$ and each filter base $\mathfrak{F}$that d-converges to x,y is an accumulation point of $F(\mathfrak{F})$ for every $y \in F(x)$.*

Proof. ($\Rightarrow$):Assume that F is l.D.sup.c. and let $\mathfrak{F} \xrightarrow{d} x$. Let W be an open set containing y, with $y \in F(x)$. Then $F(x) \cap W \neq \emptyset$, $x \in F^-(W)$ and $F^-(W)$ is d-open. Let H be an open F_σ-set such that $x \in H \subset F^-(W)$. Since $\mathfrak{F} \xrightarrow{d} x$, there exists $U \in \mathfrak{F}$such that $U \subset H$. Let $F(A) \in F(\mathfrak{F})$. Then for $A, U \in \mathfrak{F}$, there is a set U_1 of $\mathfrak{F}$such that $U_1 \subset A \cap U$. If $x \in U_1$, then since $U_1 \subset U \subset H, F(x) \cap W \neq \emptyset$. On the other hand if $x \in A$, then since $F(x) \subset F(A), F(U_1) \subset F(A)$ and since $F(U_1) \cap W \neq \emptyset, F(A) \cap W \neq \emptyset$. Thus y is an accumulation point of $F(\mathfrak{F})$.

($\Leftarrow$):Converseky, Let W be an open subset of Y containing $F(x)$. Now, the filter base $\aleph_x$ consisting of all open F_σ-set containing x d-converges to x. If F is not l.D.sup.c. at x, then there is a point $x' \in U$ for every $U \in \mathfrak{F}$ such that $F(x') \cap W = \emptyset$.If we define $\widetilde{U} = \{x' \in U \mid F(x') \cap W = \emptyset\}$, then $\widetilde{\mathfrak{F}} = \{\widetilde{U} : U \in \mathfrak{F}\}$ is a filter such that d-converges to x, since $\widetilde{U} \subset U$. Thus by hypothesis for each $y \in F(x)$, y is an accumulation point of $F(\mathfrak{F})$. But for every $\widetilde{U} \in \widetilde{\mathfrak{F}}$, $F(\widetilde{U}) \cap W = \emptyset$.This is a centradiction to hypothesis.Hence F is l.D.sup.c. at x. □

Theorem 6. *If $F : X \rightsquigarrow Y$ is u.D.sup.c.(l.D.sup.c.) and $F(X)$ is endowed with subspace topology, then $F : X \rightsquigarrow F(X)$ is u.D.sup.c.(l.D.sup.c.)*

Proof. Since $F : X \rightsquigarrow Y$ is u.D.sup.c.(l.D.sup.c.), for every open subset V of Y, $F^+(V \cap F(X)) = F^+(V) \cap F^+(F(X)) = F^+(V)(F^-(V \cap F(X)) = F^-(V) \cap F(F(X)) = F^-(V))$ is d-open. Hence $F.X \rightsquigarrow F(X)$ is u.D.sup.c.(l.D.sup.c.) □

Theorem 7. *If $F : X \rightsquigarrow Y$ is u.D.sup.c.(l.D.sup.c.) and $G : Y \rightsquigarrow Z$ u.s.c.(l.s.c.), then $G \circ F$ is u.D.sup.c(l.D.sup.c.).*

Proof. Let V be an open subset of Z. Then since G is u.s.c.(l.s.c.) $G^+(V)(G^-(V))$ is open subset of Y and since F is u.D.sup.c.(l.D.sup.c.) $F^+(G^+(V))(F^-(G^-(V)))$ is d-open in X. Thus $G \circ F$ is u.D.sup.c. (l.D.sup. c.). □

Theorem 8. *Let $\{F_\alpha : X \rightsquigarrow X_\alpha, \alpha \in \Delta\}$be a family of multifunctions and let $F : X \rightsquigarrow \prod_{\alpha \in \Delta} X_\alpha$ be defined by $F(x) = (F_\alpha(x))$. Then F is u.D.sup.c. if and only if each $F_\alpha : X \rightsquigarrow X_\alpha$ is u.D.sup.c.*

Proof. ($\Rightarrow$):Let G_{α_0} be an open set of X_{α_0}. Then $(P_{\alpha_0} \circ F)^+(G_{\alpha_0}) = F^+(P^+_{\alpha_0}(G_{\alpha_0})) = F^+(G_{\alpha_0} \times \prod_{\alpha \neq \alpha_0} X_\alpha)$. Since F is u.D.sup.c. $F^+(G_{\alpha_0} \times$

$\prod\limits_{\alpha\neq\alpha_0} X_\alpha$) is d-open in X. Thus $P_{\alpha_0}\circ F = F_\alpha$ is u.D.sup.c.. Here P_α denotes the projection of X onto α- coordinate space X_α.

($\Leftarrow$):Conversely, suppose that each $F_\alpha : X \rightsquigarrow X_\alpha$ is u.D.sup.c.. To show that multifunction F is u.D.sup.c., in view of Theorem(1) it is sufficent to show that $F^+(V)$ is d-open for each open set V in the product space $\prod\limits_{\alpha\in\Delta} X_\alpha$. Since the finite intesections and arbitary of d-open are d-open, it suffices to prove that $F^+(S)$ is d-open for every subbasic open set S in the product space $\prod\limits_{\alpha\in\Delta} X_\alpha$. Let $U_\beta \times \prod\limits_{\alpha\neq\beta} X_\alpha$ be a subbasic open set in $\prod\limits_{\alpha\in\Delta} X_\alpha$. Then $F^+(U_\beta \times \prod\limits_{\alpha\neq\beta} X_\alpha) = F^+(P_\beta^+(U_\beta)) = F_\beta^+(U_\beta)$ is d-open. Hence F is u.D.sup.c.. □

Theorem 9. *Let $F : X \rightsquigarrow Y$ be a multifuntion and $G : X \rightsquigarrow X \times Y$ defined by $G(x) = (x, F(x))$ for each $x \in X$ be the graph function. Then G is u.D.sup.c. if and only if F is u.D.sup.c. and X is D-regular.*

Proof. ($\Rightarrow$):To prove necessity, suppose that G is u.D.sup.c.. By Theorem (6) $F = P_Y \circ G$ is u.D.sup.c. where P_Y is the projection from $X\times Y$onto Y. Let U be any open set in X and let $U \times Y$ be an open set containing $G(x)$. Since G is u.D.sup.c., there exists an open F_σ-set W containing x such that the implication $x' \in W \Rightarrow G(x') \subset U \times Y$ holds. Thus $x \in W \subset U$, which shows that U is d-open and so X is D-regular.

($\Leftarrow$):To prove sufficiency, let $x \in X$ and let W be an open set containing $G(x)$. There exists open sets $U \subset X$ and $V \subset Y$ such that $(x, F(x)) \subset U \times V \subset W$. Since X is D-regular, there exists an open F_σ-set G_1 in X containing x such that $x \in G_1 \subset V$. Since F is u.D.sup.c., there exists an open F_σ-set G_2 in X containing x such that the inlication $x' \in G_2 \Rightarrow F(x') \subset V$. Let $G_1 \cap G_2 = H$. Then H is an open F_σ-set containing x and $G(H) \subset U \times V \subset W$ which implies that G is u.D.sup.c.. □

Definition 6. *Let $F : X \rightarrow Y$ be a multi function.*

a)F is said to be upper D-continous (briefly u.D.c.) at $x_0 \in X$,if for each open F_σ-set V with $F(x_0) \subset V$,there exists an open neighborhood U_{x_0} of x_0 such that the implication $x \in U_{x_0} \Rightarrow F(x) \subset V$ is hold.

b)F is said to be lower D-continuous (briefly l.D.c.) at $x_0 \in X$,if for each open F_σ-set V with $F(x_0) \cap V \neq \emptyset$ there exists an open neighborhood U_{x_0} of x_0 such that the implication $x \in U_{x_0} \Rightarrow F(x_0) \cap V \neq \emptyset$ is hold.

c)F is said to be D-continuous (briefly D.c.) at $x_0 \in X$,if it is both u.D.c. and l.D.c. at $x_0 \in X$.

d)F is said to be u.D.c. (l.D.c.,D.c.) on X,if it has this property at each point $x \in X$.

Lemma 1. *For a multifunction $F : X \rightsquigarrow Y$, the following statements are equivalent.*

(a)F is u.D.c.
(b)$F(\overline{A}) \subset [F(A)]_d$ for all $A \subseteq X$
(c)$\overline{F^+(B)} \subseteq F([(B)]_d)$ for all $B \subseteq X$
(d)For every d-closed set $K \subseteq Y, F^+(K)$ is closed
(e)For every d-open set $G \subseteq Y, F^+(G)$ is open

Proof. (a)⇒(b):Let $y \in F(\overline{A})$. Choose $x \in \overline{A}$ such that $y \in F(x)$. Let V be an open F_σ-set containing $F(x)$ so y. Since F is u.D.c., $F^+(V)$ is an open set containing x. This gives? $F^+(V) \cap A \neq \emptyset$ which in turn implies that $V \cap F(A) \neq \emptyset$ and consequently $y \in [F(A)]_d$. Hence $F(\overline{A}) \subset [F(A)]_d$.

(b)⇒(c):Let B be any subset of Y. Then $F(\overline{F^+(B)}) \subseteq [F(F^+(B))]_d \subseteq [B]_d$ and consequently $\overline{F^+(B)} \subseteq F([(B)]_d)$.

(c)⇒(d):Since a set K is d-closed if and only if $K = [K]_d$, therefore the imlication (c)⇒(d) is obvious.

(d)⇒(e):Obvious.

(e)⇒(a):'This is immediate since every open F_σ-set is d-open and since a multifunction is u.D.c. if and only if for every open F_σ-set $V, F^+(V)$ is open. □

Theorem 10. *Let X,Y and Z be topological spaces and let the function $F : X \rightsquigarrow Y$ be u.D.c. and $G : Y \rightsquigarrow Z$ be u.D.sup.c.. Then $G \circ F : X \rightsquigarrow Z$ is u.s.c.*

Proof. $[(G \circ F)^+(V) = F^+(G^+(V))]$ It is immediate in view of lemma and Theorem(1). □

D-REGULAR, D-NORMAL AND G_δ–REGULAR SPACES

Let (X, τ) be a topological space. Since the intersection of two open F_σ-sets is an open F_σ-set, the collection of all open F_σ-subsets of (X, τ) is a base for a topology τ^* on X It is immediate that a space (X, τ) is D-regular iff $\tau^* = \tau$ [Kohli,D–continuous Functions,1990].

Definition 7 (3). *A space X is said to be a G_δ-regular if for every closed G_δ-set K and a point $x \notin K$, there exist disjoint open sets U and V such that $K \subset U$ and $x \in V$.*

Definition 8. *A space X is said to be a D-normal if for every disjoint $d-$closed sets K and H, there exist disjoint open sets U and V such that $K \subset U$ and $H \subset V$.*

Theorem 11. *The multifunction $F : (X,\tau) \rightsquigarrow (Y,\gamma)$ is upper-lower D-super continuous if and only if $F : (X,\tau^*) \rightsquigarrow (Y,\gamma)$ is upper -lower semi continuous.*

Proof. ($\Rightarrow$):Let V be an open set in Y. Then since F is upper lower D-super continuous $F^+(V)(F^-(V))$ is d-open in X. So there is an open F_σ-set in X such that $U \subset F^+(V)(U \subset F^-(V))$. Hence $F^+(V) \in \tau^*(F^-(V) \in \tau^*)$ and F is upper-lower semi continuous.

($\Leftarrow$):Let V be an open set in Y. Than $F^+(V)(F^-(V))$ is open in X and since F is upper-lower semi continuous and $F^+(V) \in \tau^*(F^-(V) \in \tau^*)$, there is an open F_σ-set U such that $U \subset F^+(V)(U \subset F^-(V))$ so $F^+(V)(F^-(V))$ is d-open. Hence F is upper-lower D-super continuous. □

Theorem 12. *Let (X,τ) be topological space.Then the following are equivalent.*

(a)(X,τ) is D-regular.

(b)Every upper-lower semi continuous multifunction from (X,τ) into a space (Y,γ) is upper-lower D-super continuous.

Proof. (a)⇒(b):Obvious

(b)⇒(a):Take $(Y,\gamma) = (X,\tau)$. Then the identity multifunction I_X on X is upper-lower semi continuous and hence upper-lower D-super continuous.Thus by Theorem(11) ?$1_X : (X,\tau^*) \to (X,\tau)$ is upper-lower semi continuous.Since $U \in \tau$ implies $1_X^{-1}(U) = U \in \tau^*, \tau \subset \tau^*$. There for $\tau = \tau^*$ and so (X,τ) is D-regular. □

Theorem 13. *Let $F : (X,\tau) \rightsquigarrow (Y,\gamma)$ be a function.Then F is upper D.c.(lower D.c.) if and only if $F : (X,\tau) \rightsquigarrow (Y,\gamma^*)$ is upper semi c.(l.s.c.).*

Proof. Obvious.. □

Theorem 14. *Let $F : X \rightsquigarrow Y$ be a l.D.*sup*.c., open multifunction from a $G_\delta-$regular space onto Y. Then Y is a regular space.*

Proof. Let A be any closed subset of Y and let $y \notin A$. Then $F^+(A) \cap F^+(y) = \emptyset$. Since F is l.D.sup.c. by Theorem (1), $F^+(A)$ is d-closed and so $F^+(A) = \bigcap_{\alpha\in\Delta} F_\alpha$, where each F_α is a closed G_δ-set. Since for each $x \in F^+(y)$, $x \notin F^+(A)$ there exists an $\alpha_\circ \in \Delta$ such that $x \notin F_{\alpha_\circ}$. By $G_\delta-$regularity of X, there exist disjoint open sets $U_x$and V_x containing $F_{\alpha_\circ}$and x respectively. Since F is open $F(U_x)$ and $F(V_x)$ are disjoint open sets containing $F(x)$ and $F(F_{\alpha_0})$ respectively. Thus $y \in F(U_x)$ and $F^+(A) \subset F_{\alpha_0}$. Hence $F(F^+(A)) \subset F(F_{\alpha_0}) \subset F(V_x)$ and $A \subset F(V_x)$ so Y is regular. □

Theorem 15. *Let X be $D-$normal space and let $F : X \rightsquigarrow Y$ be a l.D.sup.c.and open surjection. Then Y is normal.*

Proof. Let $K_1$and K_2 be two disjoint closed subsets of Y. Since F is $l.D$.sup.c., then $F^+(K_1)$ and $F^+(K_2)$ are $d-$closed subsets of X. Since X is D-normal there exist two disjoint open sets U and V containing $F^+(K_1)$ and $F^+(K_2)$ respectively such that $F^+(K_1) \subset U$ and $F^+(K_2) \subset V$. Thus, $K_1 \subset F(U)$, $K_2 \subset F(V)$ and since F is open $F(U)$ and $F(V)$ are disjoint open sets containing K_1 and K_2 respectively. Hence Y is normal. □

Theorem 16. *Let $F : X \rightsquigarrow Y$ be a l.D.sup.c.and surjection defined on a $G_\delta-$regular space X. Then Y is a $G_\delta-$regular space.*

Proof. Let K be a closed $G_\delta-$set and let $y \notin K$. Then since F is l.D.sup.c., $F^+(K)$ is $d-$closed in X.

Since $x \notin F^+(K)$,for each $x \in F^-(y)$,there exists an open set G containing x such that $G \cap F^+(K) = \emptyset$. Now $X - G$ is a closed $G_\delta-$set in X. Since $x \notin X - G$ and G_δ-regularity of X,there exist two disjoint open sets U and V containing x and $X - G$ respectively such that $x \in U$ and $X - G \subset V$. Thus $F(x) \subset F(U), F(F^+(K)) \subset F(V)$ and $F(U) \cap F(V) = \emptyset$. Since $y \in F(x)$, $y \in F(U)$ and $K \subset F(v)$. Hence Y is $G_\delta-$regular. □

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