

STOCK MARKET VOLATILITY FORECASTING AND PERFORMANCE EVALUATION

Yrd. Doç. Dr. Vedat SARIKOVANLIK
I.Ü. İşletme Fakültesi

ABSTRACT

The study analyzes the forecasting performance of six statistical models on six different country stock markets. The random walk, historical mean, simple regression, generalized autoregressive conditional heteroskedasticity, (GARCH 1,1), smoothing and moving average models are employed to AEX, CAC40, DAX, İMKB, FTSE, SMI equity market data. Study range covers the period from September 17, 1997 to August 17, 2007. The findings indicate that moving average and smoothing methods are superior to other methods. The GARCH (1,1) model ranks among the poorest methods.

Key words: volatility, forecasting, performance evaluation, stock markets

HİSSE SENEDİ PİYASALARINDA VOLATİLİTE TAHMİNİ VE PERFORMANS DEĞERLENDİRMESİ

ÖZET

Çalışma, çeşitli istatistik modellerin altı farklı ülke endeksi için tahmin performansını test etme amaçlıdır. Rassal yürüyüş, tarihi ortalama, basit regresyon, düzeltme (smoothing) ve hareketli ortalama ve GARCH (1,1) teknikleriyle AEX, CAC40, DAX, İMKB, FTSE, ve SMI endeks verileri üzerinde modeller oluşturulmuş ve bu tekniklerin gelecek tahminlerinde ne kadar başarılı oldukları ampirik bulgular ışığında ortaya konulmaya çalışılmıştır. 1997-2007 dönemini kapsayan çalışmanın sonuçları, en başarılı model olarak hareketli ortalama ve düzeltme tekniklerini öne çıkarırken, değişken varyansı hesaba katan GARCH (1,1) modeli en zayıf sonuç veren modeller arasında çıkmıştır.

Anahtar Kelimeler: volatilité, tahmin, performans değérlendirmesi, hisse senedi piyasaları

INTRODUCTION

Return volatility forecasting has been a growing concern for academics and market participants during the last two decades. Volatility has become a popular risk measure after when Sharpe introduced the Sharpe ratio approach-- excess return per unit of risk-- where risk has been calculated as *volatility*¹. Since then, volatility has become a traditional and popular risk measure in the market.

Its statistical properties are well known and it feeds into several frameworks, such as modern portfolio theory and the option pricing model.² In the option pricing model, however, researchers focus only on current data and disregard past data, and hence they prefer to implement implied volatility estimates, not the traditional volatility estimates that we will concentrate in this study. According to implied volatility approach, volatile changes are believed to be driven by a stochastic process.

Therefore, the implied volatility estimates cannot be used as diagnostics or selection criteria for traditional forecasting volatility models³. Finally, sharp volatility changes in financial markets might have great effects on the economy. For example, the sudden and deep volatility of foreign exchange currency in Turkey, which is triggered by a political crisis during the 2001, had a huge impact on trade, inflation, and unemployment. On the other hand, 1987 Stock market crash in the USA, led a decrease in consumer spending⁴.

The traditional approach, as a forecast for future volatility, is based on the sample variance of historical data. Akgiray (1989), using daily U.S. stock data from January 1963 to December 1986 show that GARCH (1,1) model outperform any other forecasting models. Andersen and Bollerslev (1997), using more recent and intra-day data reach the same conclusion. Bera and Higgis (1997) report the superiority of autoregressive models for

¹ Jun, Y., "Forecasting volatility in the New Zealand stock market", *Applied Financial Economics*, 12, 193-202, 2002

² In the Black-Scholes formula, the price of an European option is a function of volatility. It is a forward-looking estimate based on market expectations.

³ Tse, S. H., K. S. Tung, "Forecasting Volatility in the Singapore Stock Market", *Asia Pacific Journal of Management*, Vol. 9, 1-13, 1992.

⁴ Garner, C. A. "Has the stock market crash reduced consumer spending?" *Financial Market Volatility and the Economy*, Federal Reserve Bank of Kansas City, 1990.

S&P500 index. When the data frequency is changed, from daily to weekly, empirical findings become mixed.

In the findings of Day-Lewis (1993), and Edey-Elliot (1992) implied volatility estimates prevail, but in more recent studies such as West-Cho (1995), Hamilton-Susmel (1994), Franses-Van Dijk (1996), Franses-Ghijssels (1999), McMillan-Speight-Gwilym (2000), ARCH class conditional and historical volatility models' superiority is strongly underlined. Dimson and Marsh, for example, apply five different models to UK stock market: random walk model, long-term mean model, moving average model, exponential smoothing model, and regression model⁵. They clearly point out that the smoothing and simple regression models perform better than other models. On the other hand, Tse (1991) and Tse-Tung (1992), using Japanese and Singaporean data find exponentially weighted moving average model outperform ARCH volatility forecasting models.

The purpose of this study is to compare 9 methods in predicting FTSE, DAX, CAC40, IMKB, SMI, AEX stock market volatilities. A considerable range of forecasting methods are employed, namely: random walk (RW), historical mean (HM), single moving average (SMA) from 4 weeks to 52 weeks, double moving average (DMA) from 4 weeks to 52 weeks, single smoothing (SS), double smoothing (DS), Holt-Winters smoothing (HWS), simple regression (REGR), and GARCH (1,1) models.

The outline of this paper is as follows. In Section 2 we describe the data, methodology and forecasting methods of the study. The methodology follows closely of what Dimson and Marsh did for U.K. market. In the third section, forecasting methods are explained and descriptive statistics are presented. The empirical findings, and finally, some conclusions are summarized in Section 4 and 5 consecutively.

1. DATA and METHODOLOGY

The study focuses on the forecasting accuracy of stock market volatility. We consider the daily and weekly closing prices of six indices covering the period from September 1997 to August 2005. The indices are: AEX Amsterdam, CAC40 Paris, DAX Frankfurt, FTSE London, IMKB Istanbul, and SMI Zurich. Emerging IMKB Istanbul stock market can be considered a contradictory data with respect to other remaining developed

⁵ Dimson, E. and P. Marsh "Volatility Forecasting Without Data Snooping", *Journal of Banking and Finance*, 14, 399-421, 1990.

stock markets. Actually, what we are trying to find out is that which volatility forecasting model is most accurate for these markets regardless of their degree of development. The study concentrates mainly on weekly index volatility forecasts. In order to figure them out we depart from compounded weekly index returns:

$$R_{w,t} = \ln (\text{Index}_{w,t} / \text{Index}_{w,t-1}) \quad (1)$$

where $R_{w,t}$ and $\text{Index}_{w,t}$ define compounded weekly return and the value of stock market index on the last day t in week w , respectively. Weekly data were chosen so as to avoid the noise and enormous fluctuations of intraday and daily data. Standard deviations of continuously compounded weekly returns are considered as the realized weekly *volatility*.

$$\sigma_{a,w} = \left(\frac{1}{n-1} \sum_{t=1}^n R_{w,t} - \mu_w^2 \right)^{0.7} \quad (2)$$

$$\mu_w = \frac{1}{n} \sum_{t=1}^n R_{w,t} \quad (3)$$

where $\sigma_{a,w}$ denotes the actual standard deviation of daily returns in week w , and μ_w is the average daily index return in week w , and n is the number of trading period.

There are 413 return volatility observations within the entire sample size, the first 207 of which are used to estimate the parameters of each forecasting models. The first part of the study is called long-run analysis, because 207 weekly estimation data are used to guess the following 206 weekly forecasts. In the second part of the study – short-run analysis, the estimation period sample size is extended until the last ten weeks, i.e., 403 observations are used to figure out the last 10 weekly forecasts. The motivation of changing the estimation period is to reflect all structural changes to forecasting period. In this case, obviously, the sample period under examination encountered several crises that created excessive volatility in the market. Some return observations are very large in absolute value and are difficult to reconcile with a normal distribution. But using the same sample size, differences between volatility forecasting at long and short horizons can be put into the test.

2. FORECASTING METHODS

We compare mainly six different volatility forecasting methods; the random walk, historical volatility, moving average of volatility, smoothing

models, simple regression, and GARCH (1,1) model. Moving average and smoothing models are re-run with 4, 8, 12, 24, 52 week data set.

Random Walk: it is the simplest possible but robust model that can exist and is defined as:

$$\sigma_{f,w} = \sigma_{a,w-1} \quad (4)$$

where, $w = 207 \dots 413$ for long run analysis and $w = 404 \dots 413$ for short run analysis; σ_f and σ_a denote forecast and actual volatilities respectively.

Hence, the random walk model assumes that the best forecast of next week's volatility is this week's actual volatility.

Historical Volatility: If the conditional expectation of volatility is assumed to be *constant*, the optimal forecast of future volatility would be the historical mean; that is

$$\sigma_{f,w} = \frac{1}{w-1} \sum_{t=1}^{w-1} \sigma_{a,t} \quad (7)$$

where, $w = 207 \dots 413$ for long run analysis and $w = 404 \dots 413$ for short run analysis. Historical mean model weights all previous data equally. The best forecast is obtained by averaging all available past observations of weekly volatility. It works well if the structure of volatility of returns is stable and if there are few or no serial correlations. More sophisticated forecast methods that capture the serial correlations in returns and *variance* of returns may prove to be superior⁶.

Moving Average: According to the historical average model, all past observations receive equal weight. In the moving average model, however, previous data are weighted more heavily than past data.

$$\sigma_{f,w} = \frac{1}{\alpha} \sum_{j=w-\alpha}^{w-1} \sigma_{a,j} \quad (5)$$

$w = 207 \dots 413$ for long run analysis and $w = 404 \dots 413$ for short run analysis. The arbitrarily chosen values of α represent 4, 8, 12, 24, 52 weeks, indicating short and long horizons. The simple moving average method is effective and efficient approach provided that the time series is stationary in both mean and variance. In computations of moving averages, different horizons are

⁶ Bollerslev, T., Chou, R.Y., Kenneth, F. K. "Arch modeling in finance: A review of the theory and empirical evidence", *Journal of Econometrics*, 52, 5-59, 1992.

chosen arbitrarily, from 4 to 52 weeks. Moreover, double moving averages for the same horizons are computed in the Eviews statistical program.

Exponential Smoothing: It is an averaging technique that uses unequal weights; however, the weights applied to past observations decline in an exponential mode. Unlike forecasts from regression models, which use fixed coefficients, forecasts from exponential smoothing methods adjust based upon past forecast errors. Single exponential smoothing forecast is given by

$$\sigma_{f,w} = (1 - \alpha) \sigma_{a,w} + \alpha \sigma_{a,w-1} \quad (7)$$

where $0 < \alpha < 1$ is the smoothing factor. By repeated substitutions, the forecasted volatility becomes the weighted average of the past values of actual volatilities, where the weights decline exponentially with time. The value of α is chosen to produce the best fit by minimizing the sum of the squared in-sample forecast errors. Single exponential smoothing is a special case of the simple moving average method, but it is more parsimonious in data usage. In the exponential smoothing and moving average models, the current forecast volatility is a function of the immediate past forecast and the immediate past observed volatility⁷. An exponential smoothing over an already smoothed time series is called double-exponential smoothing, which can capture linear trends. In the Holt-Winters' forecasting technique, short term fluctuations, which are associated with the business cycle are deprived and long-term trends are revealed.

Simple Regression: Using the regression parameters, simple autoregression models on their own lagged values give the current forecast volatility:

$$\sigma_{f,w} = \alpha + \beta \sigma_{a,w-1} \quad (8)$$

In order to obtain the forecasting coefficients, the above regression equations are re-run 206 times for the long-term forecasting period and 10 times the short-term forecasting period.

GARCH (1,1) model: In a daily GARCH (1,1) model, the conditional volatility today depends on yesterday's conditional volatility and yesterday's squared forecast error.

$$h_t = \alpha_0 + \alpha_1 e^2_{t-1} + \beta_1 h_{t-1} \quad (9)$$

The advantage of using a GARCH (1,1) model is that it is parsimonious in the number of parameters and it permits the conditional variances to depend

⁷ Dimson, E. And P. Marsh, "Volatility Forecasting", Financial Markets, Institutions & Instruments, 6, Number 1, 1990.

on past-realized variances, which is consistent with the observed volatility pattern of the stock markets. It is obvious that GARCH models require demanding computational efforts.

Summary statistics for the entire sample period, the estimation period and forecasting period are shown in Tables 1-A and 1-B.

Estimation	AEX	CAC40	DAX	FTSE	IMKB	SMI
Mean	0.0125	0.0131	0.0140	0.0113	0.0323	0.0110
Median	0.0106	0.0117	0.0128	0.0104	0.0291	0.0095
Maximum	0.0434	0.0421	0.0427	0.0306	0.1345	0.0438
Minimum	0.0000	0.0017	0.0000	0.0000	0.0000	0.0017
Std. Dev.	0.0071	0.0065	0.0072	0.0052	0.0206	0.0066
Skewness	1.7339	1.3280	0.9459	0.6914	1.7319	1.7720
Kurtosis	6.8567	5.9575	4.0204	3.2854	8.1133	7.2150
Forecast						
Mean	0.0146	0.0135	0.0156	0.0101	0.0201	0.0109
Median	0.0108	0.0101	0.0122	0.0074	0.0176	0.0084
Maximum	0.0616	0.0497	0.0659	0.0435	0.0823	0.0447
Minimum	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Std. Dev.	0.0110	0.0093	0.0109	0.0076	0.0118	0.0078
Skewness	1.4247	1.4229	1.4635	1.7620	1.6263	1.5872
Kurtosis	4.8055	4.6441	5.4167	6.4664	7.7145	5.7465
Entire						
Mean	0.0135	0.0133	0.0148	0.0107	0.0262	0.0109
Median	0.0107	0.0112	0.0124	0.0090	0.0227	0.0090
Maximum	0.0616	0.0497	0.0659	0.0435	0.1345	0.0447
Minimum	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Std. Dev.	0.0093	0.0080	0.0093	0.0066	0.0179	0.0072
Skewness	1.6916	1.4820	1.5155	1.4573	2.0156	1.6721
Kurtosis	6.2545	5.5369	6.3089	6.1427	10.0046	6.4104

Table 1a: Long-Run Volatility

Table 1b: Short-Run Volatility

Estimation	AEX	CAC40	DAX	FTSE	IMKB	SMI
Mean	0.0137	0.0134	0.0150	0.0108	0.0265	0.0110
Median	0.0108	0.0113	0.0128	0.0091	0.0229	0.0091
Maximum	0.0616	0.0497	0.0658	0.0434	0.1344	0.0447
Minimum	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Std. Dev.	0.0093	0.0080	0.0093	0.0065	0.0178	0.0072
Skewness	1.6725	1.4632	1.4960	1.4394	2.0123	1.6521
Kurtosis	6.1672	5.4700	6.2508	6.0920	9.9982	6.3288
Forecast						
Mean	0.0063	0.0067	0.0070	0.0052	0.0097	0.0051
Median	0.0058	0.0068	0.0071	0.0048	0.0093	0.0055
Maximum	0.0108	0.0120	0.0121	0.0103	0.0158	0.0072
Minimum	0.0015	0.0036	0.0034	0.0031	0.0038	0.0022
Std. Dev.	0.0028	0.0024	0.0023	0.0021	0.0036	0.0017
Skewness	0.2202	0.6735	0.6555	1.3386	-0.0011	-0.420
Kurtosis	2.5046	3.1778	3.6769	4.2261	2.3521	1.9013
Entire						
Mean	0.0135	0.0133	0.0148	0.0107	0.0262	0.0109
Median	0.0107	0.0112	0.0124	0.0090	0.0227	0.0090
Maximum	0.0616	0.0497	0.0659	0.0435	0.1345	0.0447
Minimum	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Std. Dev.	0.0093	0.0080	0.0093	0.0066	0.0179	0.0072
Skewness	1.6916	1.4820	1.5155	1.4573	2.0156	1.6721
Kurtosis	6.2545	5.5369	6.3089	6.1427	10.004	6.4104

The *average* standard deviation for the entire period for all indices is less than the forecast period and greater than the estimation period for long-run analysis. This is to say that investors become more risk averse in the second half of the sample, which is exactly right after 9/11/2001. Average standard deviations figures shown in above tables correspond to annualized volatilities of 7.7 % and 18.9 %. Positive skewness is evident throughout the sample. Kurtosis statistics, meanwhile, range between 5.53 and 10.01 for the entire period, which is strong evidence that extremes are more substantial than would be expected from a normal random variable. On the other hand, when we plot the weekly standard deviations of the indices in Figure 1, we note almost the same volatility clustering in all stock markets, except IMKB, Istanbul. Peculiar internal dynamics of IMKB, during this period, might have been practiced differently than other stock markets.

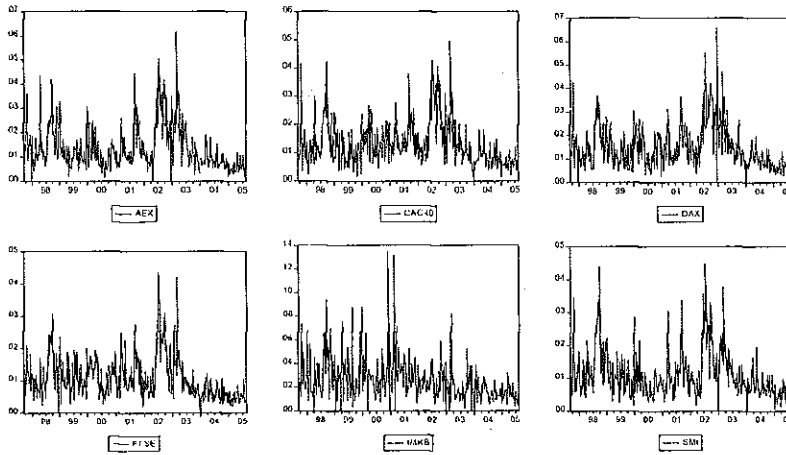


Figure 1: Stock Market Weekly Standard Deviations

3. EMPIRICAL RESULTS

In evaluating model forecasting performance, the most popular accuracy measures⁸, MAPE and RMSE were used for measuring out of sample accuracy.

$$RMSE = \frac{1}{206} \sum_{t=207}^{413} ((S_{t,w} - S_{a,w})^2)^{0.5} \quad (10)$$

$$MAPE = \frac{1}{206} \sum_{t=207}^{413} \left| \frac{(S_{f,w} - S_{a,w})}{S_{a,w}} \right| \quad (11)$$

The forecast errors are then compared with actual values using root mean squared error (RMSE) and mean absolute percentage error (MAPE). Their calculations are shown above. The method with the lowest forecast error will be the best predictor of return volatility across particular time interval. The relative values of each forecast method are also compared to the worst method, by normalizing the error of the worst method to a value of 1.00 and using it as a benchmark to assess the rest. Hence, the lowest relative

⁸ Makridakis, S., Wheelwright, S., Hyndman, R. "Forecasting Methods and Applications", John Wiley & Sons, 3rd edition, 1998.

value will signal the best forecast method⁹. The results are presented in Tables 3 to 6.

In forecasting weekly standard deviations, the superiority of the moving average and smoothing methods over the rest of models, specifically over the GARCH (1,1) models is evident for all indices. Both short-run and long-run forecasting results show similar patterns.

On the basis of RMSE and MAPE evaluation criteria, the historical mean, random walk, GARCH (1,1) and regression models usually yield larger errors, hence they do not give accurate future volatility forecasts for the stated data range.

Turning our attention to the statistics of RMSE and MAPE, a number of key features can be identified from Table 3 and 4. Out of 6 indices, single moving average forecasting method produces the smallest error statistics for *cac40*, *dax*, *ftse*, and *smi* indices. The smoothing forecasting methods, however, dominate the forecasting performance for both *AEX* and *IMKB* indices. Finally, the historical mean approach provide clearly the least accurate volatility forecasts.

⁹ D. M. Walsh, G.Y.G. Glenn, "Forecasting Index Volatility: sampling interval and non-trading effects", *Applied Financial Economics*, 8, 477-485, 1998.

Table 3a: RMSE statistics – Long Run Forecasting

	<i>AEX</i>			<i>CAC40</i>			<i>DAX</i>		
	<i>Actual</i>	<i>Relative</i>	<i>Rank</i>	<i>Actual</i>	<i>Relative</i>	<i>Rank</i>	<i>Actual</i>	<i>Relative</i>	<i>Rank</i>
RW	0.839249	0.654132	11	0.754389	0.668744	14	0.920232	0.699364	14
HM	1.282995	1.000000	17	1.128067	1.000000	17	1.315813	1.000000	17
SMA4	0.554632	0.432295	4	0.48636	0.431145	1	0.585250	0.444782	1
SMA8	0.678472	0.528819	5	0.56541	0.501220	2	0.645925	0.490894	2
SMA12	0.777245	0.605805	7	0.615498	0.545622	5	0.682779	0.518903	4
SMA24	0.965708	0.752698	15	0.708662	0.628209	12	0.785546	0.597004	12
SMA52	1.167531	0.910004	16	0.814058	0.721639	16	0.967527	0.735307	16
DMA4	0.685965	0.534659	6	0.580661	0.514739	3	0.673972	0.512210	3
DMA8	0.821811	0.640542	9	0.644166	0.571035	9	0.697780	0.530304	5
DMA12	0.897506	0.699540	13	0.669131	0.593166	11	0.726016	0.551762	7
DMA24	0.947585	0.738573	14	0.639317	0.566737	8	0.750529	0.570392	11
DMA52	0.797055	0.621246	8	0.604547	0.535914	4	0.746191	0.567095	10
SMS	0.098186	0.076529	2	0.621028	0.550524	6	0.726226	0.551922	8
SMD	0.098033	0.076409	1	0.644851	0.571642	10	0.741652	0.563645	9
SMHW	0.098547	0.076810	3	0.621212	0.550687	7	0.725236	0.551169	6
REG	0.835138	0.650929	10	0.755387	0.669629	15	0.939213	0.713789	15
GARCH	0.869231	0.677502	12	0.739511	0.655556	13	0.841493	0.639523	13

Table 3b: RMSE statistics – Long Run Forecasting

	<i>FTSE</i>			<i>IMKB</i>			<i>SMI</i>		
	<i>Actual</i>	<i>Relative</i>	<i>Rank</i>	<i>Actual</i>	<i>Relative</i>	<i>Rank</i>	<i>Actual</i>	<i>Relative</i>	<i>Rank</i>
RW	0.567513	0.600881	14	1.68463	0.824075	15	0.633925	0.234939	14
HM	0.944469	1.000000	17	2.044269	1.000000	17	0.919859	0.340910	16
SMA4	0.378019	0.400245	1	1.024523	0.501169	2	0.415971	0.154163	1
SMA8	0.445000	0.471164	2	1.169546	0.572110	4	0.498794	0.184859	3
SMA12	0.478729	0.506877	5	1.238986	0.606078	7	0.539634	0.199994	6
SMA24	0.551754	0.584195	13	1.261803	0.617239	8	0.623326	0.231011	13
SMA52	0.639487	0.677087	16	1.280037	0.626159	10	0.678499	0.251459	15
DMA4	0.460301	0.487365	4	1.205761	0.589825	5	0.516704	0.191496	4
DMA8	0.498338	0.527638	9	1.285213	0.628691	11	0.566074	0.209793	9
DMA12	0.516161	0.546509	11	1.232954	0.603127	6	0.589931	0.218635	11
DMA24	0.494650	0.523734	8	1.067232	0.522061	3	0.53884	0.199700	5
DMA52	0.458750	0.485722	3	0.810852	0.396646	1	0.480382	0.178035	2
SMS	0.485841	0.514407	7	1.294444	0.633206	13	0.542926	0.201214	8
SMD	0.505641	0.535371	10	1.267749	0.620148	9	0.566287	0.209872	10
SMHW	0.485503	0.514049	6	1.290142	0.631102	12	0.542762	0.201153	7
REG	0.569656	0.603149	15	1.560275	0.763243	14	0.604193	0.223920	12
GARCH	0.531410	0.562654	12	1.723819	0.843245	16	2.69825	1.000000	17

Table 4a: MAPE statistics – Long Run Forecasting

	<i>AEX</i>			<i>CAC40</i>			<i>DAX</i>		
	<i>Actual</i>	<i>Relative</i>	<i>Rank</i>	<i>Actual</i>	<i>Relative</i>	<i>Rank</i>	<i>Actual</i>	<i>Relative</i>	<i>Rank</i>
RW	432.5305	0.087925	11	48.31287	0.453541	12	40.57148	0.413585	11
HM	378.1778	0.076876	8	106.5238	1.000000	17	98.09715	1.000000	17
SMA4	318.3754	0.064719	5	33.93025	0.318523	1	29.40854	0.299790	1
SMA8	343.3322	0.069793	6	37.82186	0.355055	2	30.68	0.312751	2
SMA12	433.6863	0.088160	12	40.26652	0.378005	4	31.87575	0.324941	3
SMA24	574.751	0.116836	16	48.239	0.452847	11	39.31291	0.400755	10
SMA52	459.7843	0.093465	14	61.95679	0.581624	16	55.50509	0.565818	15
DMA4	353.9183	0.071945	7	40.25116	0.377861	3	33.41755	0.340658	5
DMA8	440.9156	0.089629	13	42.29952	0.397090	5	32.88991	0.335279	4
DMA12	495.4897	0.100723	15	44.86318	0.421156	9	35.9134	0.366100	9
DMA24	388.4948	0.078973	9	47.83062	0.449013	10	42.38252	0.432046	13
DMA52	142.6673	0.029001	4	53.69042	0.504023	13	52.81526	0.538398	14
SMS	7.195435	0.001463	1	43.34362	0.406891	7	35.09614	0.357769	7
SMD	7.195996	0.001463	2	44.2508	0.415407	8	35.37967	0.360660	8
SMHW	7.247603	0.001473	3	42.76529	0.401462	6	33.7357	0.343901	6
REG	406.5851	0.082651	10	60.88511	0.571563	15	58.32734	0.594588	16
GARCH	4919.317	1.000000	17	56.20082	0.527589	14	41.71869	0.425279	12

Table 4b: MAPE statistics – Long Run Forecasting

	<i>FTSE</i>			<i>IMKB</i>			<i>SMI</i>		
	<i>Actual</i>	<i>Relative</i>	<i>Rank</i>	<i>Actual</i>	<i>Relative</i>	<i>Rank</i>	<i>Actual</i>	<i>Relative</i>	<i>Rank</i>
RW	46.47661	0.369305	10	74.29848	0.506849	9	56.1322	0.469336	11
HM	125.8489	1.000000	17	146.5889	1.000000	17	119.5992	1.000000	17
SMA4	33.8349	0.268853	1	57.29208	0.390835	2	38.4683	0.321644	1
SMA8	39.41902	0.313225	2	66.58059	0.454199	5	46.8138	0.391422	2
SMA12	41.80387	0.332175	3	75.11508	0.512420	10	49.29418	0.412162	3
SMA24	49.31204	0.391835	13	74.21762	0.506298	8	58.1979	0.486608	13
SMA52	65.75438	0.522487	16	80.62536	0.550010	14	71.6329	0.598941	16
DMA4	41.90123	0.332949	4	68.58816	0.467895	6	49.46078	0.413555	4
DMA8	43.55615	0.346099	6	77.10902	0.526022	11	51.19554	0.428059	6
DMA12	45.51735	0.361682	9	71.87643	0.490327	7	53.81831	0.449989	9
DMA24	49.14995	0.390547	12	61.66219	0.420647	3	56.48699	0.472303	12
DMA52	56.75507	0.450978	14	45.28123	0.308899	1	60.47747	0.505668	14
SMS	43.91314	0.348935	7	80.02919	0.545943	13	51.26921	0.428675	7
SMD	44.37249	0.352585	8	66.0381	0.450499	4	52.17666	0.436263	8
SMHW	42.28312	0.335983	5	78.03297	0.532325	12	49.81728	0.416535	5
REG	61.57592	0.489284	15	108.6497	0.741186	16	66.44327	0.555550	15
GARCH	47.70597	0.379073	11	101.3328	0.691272	15	55.5269	0.464275	10

Table 5a: RMSE statistics – Short Run Forecasting

	AEX			CAC40			DAX		
	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank
RW	0.004115	0.433226	14	0.004620	0.577487	15	0.003894	0.402646	14
HM	0.009499	1.000000	17	0.008000	1.000000	17	0.009670	1.000000	17
SMA4	0.002773	0.291889	2	0.002902	0.362799	8	0.002800	0.289533	7
SMA8	0.002771	0.291757	1	0.002612	0.326491	2	0.002609	0.269794	2
SMA12	0.002829	0.297826	3	0.002591	0.323876	1	0.002565	0.265288	1
SMA24	0.002912	0.306525	6	0.002737	0.342182	3	0.002692	0.278411	4
SMA52	0.003005	0.316333	7	0.002753	0.344086	4	0.002757	0.285121	5
DMA4	0.003202	0.337056	13	0.003064	0.383045	9	0.003002	0.310490	9
DMA8	0.003173	0.334027	10	0.002778	0.347302	7	0.002803	0.289818	8
DMA12	0.003080	0.324247	8	0.002759	0.344863	5	0.002759	0.285293	6
DMA24	0.002834	0.298378	4	0.002768	0.345978	6	0.002692	0.278357	3
DMA52	0.006672	0.702401	16	0.005061	0.632600	16	0.005495	0.568253	16
SMS	0.003143	0.330848	9	0.003355	0.419446	10	0.003105	0.321076	11
SMD	0.002853	0.300387	5	0.003437	0.429588	12	0.003154	0.326199	12
SMHW	0.003184	0.335206	11	0.003371	0.421443	11	0.003063	0.316796	10
REG	0.004613	0.485619	15	0.004381	0.547667	14	0.004713	0.487433	15
GARCH	0.003195	0.336385	12	0.004000	0.499950	13	0.003483	0.360198	13

Table 5b: RMSE statistics – Short Run Forecasting

	FTSE			IMKB			SMI		
	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank
RW	0.002608	0.369349	11	0.006927	0.361811	10	0.002509	0.158784	13
HM	0.007061	1.000000	17	0.019145	1.000000	17	0.006941	0.439300	16
SMA4	0.002269	0.321354	2	0.003719	0.194271	1	0.001624	0.102778	2
SMA8	0.002470	0.349769	7	0.003853	0.201258	2	0.001693	0.107173	6
SMA12	0.002215	0.313717	1	0.004757	0.248456	5	0.001595	0.100921	1
SMA24	0.002370	0.335686	5	0.006998	0.365517	11	0.001685	0.106625	4
SMA52	0.002308	0.326861	3	0.006525	0.340838	8	0.001847	0.116921	9
DMA4	0.002837	0.401746	13	0.004044	0.211214	4	0.001775	0.112344	8
DMA8	0.002475	0.350503	8	0.005577	0.291324	6	0.001689	0.106882	5
DMA12	0.002383	0.337438	6	0.007668	0.400551	13	0.001699	0.107511	7
DMA24	0.002323	0.328988	4	0.007274	0.379924	12	0.001675	0.105986	3
DMA52	0.003040	0.430475	15	0.008408	0.439174	15	0.004111	0.260213	15
SMS	0.002665	0.377396	12	0.006586	0.344007	9	0.002105	0.133227	10
SMD	0.002837	0.401758	14	0.004003	0.209089	3	0.002146	0.135808	12
SMHW	0.002592	0.367120	10	0.006041	0.315555	7	0.002122	0.134275	11
REG	0.003058	0.433037	16	0.010311	0.538581	16	0.002659	0.168264	14
GARCH	0.002486	0.352128	9	0.008032	0.419551	14	0.015800	1.000000	17

Table 6: MAPE statistics – Short Run Forecasting

	<i>AEX</i>			<i>CAC40</i>			<i>DAX</i>		
	<i>Actual</i>	<i>Relative</i>	<i>Rank</i>	<i>Actual</i>	<i>Relative</i>	<i>Rank</i>	<i>Actual</i>	<i>Relative</i>	<i>Rank</i>
RW	0.124730	0.389518	14	0.093494	0.520734	16	0.055446	0.259219	10
HM	0.320216	1.000000	17	0.179543	1.000000	17	0.213896	1.000000	17
SMA4	0.092276	0.288169	5	0.057376	0.319569	9	0.053168	0.248568	8
SMA8	0.092076	0.287543	4	0.048221	0.268575	3	0.049201	0.230021	5
SMA12	0.094084	0.293813	6	0.046695	0.260080	1	0.046553	0.217646	2
SMA24	0.086776	0.270991	3	0.048047	0.267610	2	0.047173	0.220541	3
SMA52	0.099464	0.310615	8	0.050029	0.278649	7	0.051346	0.240052	7
DMA4	0.108275	0.338132	13	0.057249	0.318858	8	0.057258	0.267689	11
DMA8	0.104461	0.326221	11	0.049063	0.273265	6	0.050315	0.235231	6
DMA12	0.094767	0.295947	7	0.048525	0.270270	5	0.048469	0.226599	4
DMA24	0.082798	0.258569	2	0.048351	0.269298	4	0.045190	0.211273	1
DMA52	0.136476	0.426201	15	0.088212	0.491314	14	0.105051	0.491131	16
SMS	0.101604	0.317297	9	0.064718	0.360462	11	0.057921	0.270793	12
SMD	0.079461	0.248148	1	0.067416	0.375488	12	0.059570	0.278499	13
SMHW	0.104139	0.325214	10	0.063864	0.355705	10	0.054685	0.255662	9
REG	0.151999	0.474675	16	0.088598	0.493467	15	0.098991	0.462801	15
GARCH	0.105752	0.330252	12	0.082444	0.459190	13	0.065694	0.307130	14

Table 6: MAPE statistics – Short Run Forecasting

	<i>FTSE</i>			<i>IMKB</i>			<i>SMI</i>		
	<i>Actual</i>	<i>Relative</i>	<i>Rank</i>	<i>Actual</i>	<i>Relative</i>	<i>Rank</i>	<i>Actual</i>	<i>Relative</i>	<i>Rank</i>
RW	0.048527	0.237043	6	0.084982	0.276705	6	0.066526	0.323920	14
HM	0.204717	1.000000	17	0.307120	1.000000	17	0.205380	1.000000	17
SMA4	0.052552	0.256705	8	0.052028	0.169405	1	0.044290	0.215650	1
SMA8	0.055902	0.273069	9	0.063237	0.205902	3	0.048220	0.234783	4
SMA12	0.046582	0.227543	2	0.081052	0.263909	5	0.046501	0.226416	2
SMA24	0.046657	0.227909	3	0.115998	0.377697	11	0.048179	0.234587	3
SMA52	0.048200	0.235447	5	0.107997	0.351645	9	0.053237	0.259213	9
DMA4	0.069067	0.337379	14	0.062870	0.204708	2	0.050911	0.247888	8
DMA8	0.049854	0.243528	7	0.094915	0.309048	7	0.049807	0.242514	7
DMA12	0.047180	0.230464	4	0.128480	0.418339	14	0.048392	0.235621	5
DMA24	0.042322	0.206732	1	0.119700	0.389750	12	0.048979	0.238479	6
DMA52	0.074228	0.362587	15	0.138702	0.451621	15	0.103767	0.505246	16
SMS	0.061399	0.299921	12	0.109728	0.357279	10	0.055352	0.269509	11
SMD	0.067556	0.329997	13	0.065351	0.212786	4	0.057464	0.279795	12
SMHW	0.056323	0.275127	10	0.100532	0.327336	8	0.054582	0.265760	10
REG	0.078163	0.381809	16	0.167144	0.544229	16	0.075743	0.368793	15
GARCH	0.057535	0.281044	11	0.128029	0.416870	13	0.065901	0.320875	13

The short run forecasting statistics are shown in Table 5, and Table 6. Without any exception, single moving average methods reflect the most accurate forecasts in RMSE statistics. With a little few differences, almost the same results are evident in MAPE statistics, as shown in Table 6. Smoothing and double moving average methods are gaining the best accurate forecast rankings in Amsterdam and London stock markets. Moreover, the gain in performance of the random walk model in London and Istanbul stock markets is highly noticeable. Finally, the performance of the remaining models, especially the simple regression, historical mean and GARCH (1,1), ranks poorly compared to remaining models.

These results are almost in line with those of Tse¹⁰, and Balaban, et al¹¹, defining smoothing method as the best predictor for weekly data. Interestingly, even in periods of excess volatility where GARCH processes are deemed to do better, the GARCH (1,1) model fails to outperform the competing methods. Contrary to the findings by Akgiray¹² the GARCH (1,1) model is by far the most inferior of the rest methods.

CONCLUSION

The study focuses on the forecasting accuracy of stock market volatility. The main purpose is to compare and evaluate various statistical and econometric models in forecasting the volatility of six different stock markets. These are: Amsterdam (*AEX*), Paris (*CAC40*), Frankfurt (*DAX*), London (*FTSE*), Istanbul (*IMKB*), and Zurich (*SMI*).

The volatility is considered as the within week standard deviation of continuously weekly realized index returns. In the study, six forecasting methods are employed – random walk, historical mean, moving average, smoothing techniques, regression model, and GARCH (1,1) model. Based on the sample data, weekly observations are used to produce long-run and short-run forecasting horizons. Out of sample forecasting accuracy has been determined with mean absolute percentage error (MAPE) and root mean squared error (RMSE) statistics.

¹⁰ Tse, Y. K., "Stock Returns Volatility in the Tokyo Stock Exchange", Japan and the World Economy, 3, 285-298, 1991.

¹¹ Balaban, E., Bayar, A., R. Faff, "Forecasting Stock Market Volatility: Evidence from Fourteen Countries", University of Edinburgh, Center For Financial Markets Research Working Paper 02.04

¹² Akgiray, V. "Conditional Heteroscedasticity in Time Series of Stock Returns: Evidence and Forecasts", *Journal of Business*, 62, 55-80, 1989.

The findings in the previous section show the superiority of the moving average method over the historical, random walk, regression and the GARCH(1,1) model in forecasting future volatility in the Amsterdam (*AEX*), Paris (*CAC40*), Frankfurt (*DAX*), London (*FTSE*), Istanbul (*IMKB*), and Zurich (*SMI*) stock markets. The GARCH (1,1) model, while the most sophisticated, ranks among the poorest methods along with historical mean, random walk and one lag regression model. The deficiency of the GARCH (1,1) model can be attributed to sample data range and frequency requirements.

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