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A stochastic model to estimate origin-destination probabilities for electricity supply in GCC

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Abstract

In this paper a stochastic model has been developed to estimate the origin-destination (OD) matrices. Particularly, the model is capable of estimating the OD probabilities for the electricity supply in the GCC. The major basis in this model is its ability to account explicitly for missing observations. The maximum likelihood estimates for the model parameters are derived in this work. In addition, the long run behavior of the model is studied. The long run mean of amount of electricity (LRAOE) enters as well as the long run mean cost (LRMC) per unit of time is considered for this model. Also, the LRAOE and the LRMC are considered for the case when one country uses all the amount of electricity enters (absorbing state) and the case where more than one country use all the amount of electricity enters (k absorbing states) conditional on one absorbing state.

Keywords: *Origin-Destination Matrices, Stochastic Model, OD Probabilities, Long Run Average Amount, Long Run Mean Constant*

Körfez Arap Ülkeleri İşbirliği Konseyi'nde elektrik dağıtım hizmeti için kaynak-menzil olasılıklarının tahminine yönelik bir stokastik model

Özet

Bu makalede, kaynak-menzil matrisinin tahminine yönelik bir stokastik model geliştirilmiştir. Bu model bilhassa, Körfez Arap Ülkeleri İşbirliği Konseyi'ne elektrik dağıtım hizmeti için kaynak-menzil olasılıklarının öngörülmesine sağlamaktadır. Bu modelin temel özelliği, kayıp gözlemleri açık bir şekilde hesaplamaya olanak vermesidir. Bu çalışmada kullanılan parametreler, en çok olabilirlik tahmin yöntemi ile elde edilmiştir. Bunun yanı sıra, modelin uzun vadede nasıl çalışacağı üzerinde çalışılmıştır. Uzun dönemde sağlanan ortalama elektrik miktarının yanı sıra uzun vadede birim zaman için geçerli ortalama maliyetler bu modelde hesaplanmıştır. Ayrıca bu iki parametre, belirli bir ülkenin sağlanan tüm elektriği tükettiği (yutucu durum) ve bir yutucu durum şartına bağlı olarak, birden fazla ülkenin bölgeye sağlanan tüm elektriği tükettiği (k adet yutucu durum) senaryolar için kullanılmıştır.

Anahtar Sözcükler: *Kaynak-Menzil Matrisleri, Stokastik Model, KM Olasılıkları, Uzun Vadeli Ortalama Değer, Uzun Vadeli Ortamala Sabit*

1. Introduction

Stochastic models have received considerable attention by many researchers in the literature due to their wide applications in many areas. Such applications can be found in business, engineering, transportation, energy, electricity, etc. Thus studying the estimation of the OD directly from the observed amount of electricity is of great importance.

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Different authors have studied the application of stochastic model in estimating the OD probabilities. For example, Alawaneh et al. [1] have studied the estimation of the OD directly from observed traffic counts. This model is capable of estimating the OD probabilities for any intersection or interchange irrespective of the number of entrances and exits. Mike [2] has studied the long-term memory in US stock market volatility. Two commonly tests, the semi-parametric and the modified re-scaled range tests are applied to various transformations of stock return series for different US companies. Walls [3] has examined the volatility, volume, and maturity effects in electricity future market. His work appears to be the first empirical investigation of the market for electricity future.

This paper focuses on estimating the OD matrices for the amount of electricity supply in the GCC and consequently the OD probabilities for the electricity supply irrespective of number of entrances and exits from countries as discussed in section 3. Section 4 considers the steady state distribution and its usage to compute the long run mean amount of electricity simply LRMAOE as well as the long run mean cost LRMC per unit of time. Section 5 considers the steady state distribution for absorbing Markov Chain and its usage to compute the LRMAOE and one absorbing state as in subsection 5.1 and for the case of the existence of r absorbing states conditional on the eventual absorption into a specific state, say 1 as in subsection 5.2. Finally, section 6 considers some applications of the above results obtained in the suggested paper.

2. Model Specification

Suppose the GCC consists of r countries. This means we have r inputs and r outputs. Let N_{ij} denotes the amount of electricity entering approach i and exiting approach j. Thus, we have (r × r) matrix with cell counts N_{ij} and $\sum_{i,j} N_{ij} = N$. Let X(t) denotes the total amount of electricity that enter the facility at time t. Then $\{X(t); t \geq 0\}$ is a discrete Markov Chain with state space $S = \{1,2,....,r\}$ and transition probability matrix:

$$P = [P_{ij}]; i, j \in S$$

with

$$P_{ij} = P(X(t) = j \mid X(t-1) = i); i, j \in S \tag{1}$$

where P_{ij} is the probability that the amount of electricity is exiting state j at time t, given that it left entrance state i at time t-1. Thus, P is given by

$$P = \begin{bmatrix} P_{11} & P_{12} & .. & .. & P_{1r} \\ P_{21} & P_{22} & .. & .. & P_{2r} \\ . & . & . & . & . \\ . & . & . & . & . \\ P_{r1} & P_{r2} & .. & .. & P_{rr} \end{bmatrix}, \tag{2}$$

where each row in the above matrix represents an entrance state and each column represents an exit state. The P_{ij} are called the one-step transition (one-step transition OD probabilities) probabilities that satisfy the following conditions:

$$1)\ P_{ij} \geq 0 \qquad for\ i, j = 1,2,..,r$$

$$2)\sum_{j=1}^{r} P_{ij} = 1 \qquad for\ i = 1,2,...,r$$

It should be noted that the P_{ij}'s completely define the Markov Chain. The analysis of Markov Chain probabilities is possible by realizations of the process. Central to these calculations are the n-step transition probability matrix P_n, where

$$P = [P_{ij}^{(n)}]; \qquad i, j \in S$$

Here $P_{ij}^{(n)}$ denotes the probability for the amount of the electricity entering state i at time m and exiting state j at time m+n, where n represents the travel time to cross the facility, i.e.,

$$P_{ij}^{(n)} = P(X_{m+n} = j \mid X_m = i),\ i, j \in S \tag{3}$$

3. Estimation of Model Parameters

Let $X_{N+1} = (x_0, x_1, ..., x_N)$ be a random sample from the discrete Markov Chain with OD probabilities P_{ij}. We use the likelihood function given by Basawa and Prakasa [4]:

$$p = \pi_0 \prod_{k=1}^{N} p_{x_{k-1}} x_k = \pi_0 \prod_{i,j=1}^{r} p_{i,j} N_{ij} \tag{4}$$

where $\pi_0 = (\pi_{1.0}, \pi_{2.0}, ..., \pi_{r.0})$ is the initial distribution and N_{ij} is the frequency of one-step transitions $i \to j$ in the sample. Following the same approach as in Alwaneh et al. [1], it is easily shown that the maximum likelihood estimate of p_{ij} is given by

$$\hat{p}_{ij} = \frac{N_{ij}}{N_i}, \qquad N_i = \sum_{j=1}^{r} N_{ij} \tag{5}$$

4. Steady State Distribution

Consider a Markov Chain $\{X_t; t \geq 0\}$, with state space $S = \{1,2,...,r\}$ and transition matrix P. Then the steady state probability (π_j) for the amount of electricity has been in supplied and is in exit state j, irrespective of the starting entrance state. Moreover, π_j gives the long run mean function of time that the process $\{X_t\}$ is in the exit state j. Note that π_j is given by solving the linear equations [5] such that

$$\pi_j = \sum_{k=1}^{r} \pi_k p_{ki} \qquad for\ i = 1,2,...,r \tag{6}$$

and $\pi_1 + \pi_2 + ... + \pi_r = 1$. Also, π_j is independent as the limiting distribution:

$$\pi_j = \lim_{n \to \infty} p_{ij}^{(n)} \tag{7}$$

Note that, if each visit to the exit state j gains an amount of electricity N_j, the steady state on the long run mean amount of electricity per unit of time (LRMAOE) associated with this Markov Chain is

$$\text{LRMAOE} = \sum_{j=1}^{r} \pi_j N_j \tag{8}$$

Now using the limiting distribution, it is easily shown that the long run mean cost (LRMC) per unit of time is given by

$$\text{LRMC} = \sum_{j=1}^{r} \pi_j C_j \tag{9}$$

where C_j is the cost incurred by each visit to state j.

5. Steady State Distribution for Absorbing Markov Chain

In this section, we derive the steady state distribution for Markov Chain with one or more absorbing states. More specifically, the absorbing state occurs if one country of GCC uses all the amount of electricity enters and has no amount exit. Also, r absorbing states (r >1) occurs if more than one country of the GCC, say r, use all the amount of electricity enters and no amount exit. Subsections 5.1 and 5.2 study the suggested cases.

5.1. Case of 1 Absorbing State

Let the Markov Chain $\{X_t\}$ has states 1, 2, 3,…, r where state 1 is an absorbing state, and the transition matrix is given by

$$P = \begin{bmatrix} 1 & 0' \\ P_o & Q \end{bmatrix}; \; P_o \neq 0 \tag{10}$$

Here Q is $(r-1)\times(r-1)$ matrix and P_o and 0 are both $(r-1)$ vectors. Assume Q is irreducible and a periodic matrix, with transient state is $T = \{2,3,....,r\}$. If the process has the initial distribution π_0 over transient states, then the probability that absorbed by time n is $\sum_{i\in T} \pi_{i0} P_{io}^{(n)}$ and given that it is still in T, the condition probability that it is in state j is given by

$$\frac{\sum_{i\in T} \pi_{oo} P_{ij}^{(n)}}{\sum_{i\in T} \pi_{i0} (1 - P_{i1}^{(n)})} \; . \tag{11}$$

Using Perron-Frobenius theory for non-negative matrices, we get [6]:

$$P_{ij}^{(n)} = \rho_1^n a_i b_j + 0(n^{m_2-1}|\rho_2|^n) \;. \tag{12}$$

where $\{a_j\}$ and $\{b_j\}; j \in T$ are elements in the normalized right and left eigenvalues of Q respectively corresponding to $\rho_1, (\sum_{j\in T} a_j b_j = 1 \& \sum_{j\in T} b_j = 1)$, and ρ_1 is the maximum modulus eigenvalue.

Note that

$$\lim_{n\to\infty} p(x_n = j \,|\, x_n \in T) = b_j$$

and

$$\lim_{n\to\infty}\lim_{m\to\infty} p(x_m = j \,|\, x_n \in T, n > m) = b_j a_j \;,$$

$\{b_j\}$ and $\{a_j b_j\}$; $j \in T$ are usually known as the quasi and product quasi stationary distributions. Thus, the steady state distribution π_j for this case is given by [7]:

$$\pi_j = b_j a_j \qquad ; \qquad j \in T \tag{13}$$

Therefore, the long run average of electricity (LRAOE) and the long run mean cost per unit of time (LRMC) are given by

$$\text{LRAOE} = \sum_{j=2}^{r} \pi_j N_j \tag{14}$$

and

$$\text{LRMC} = \sum_{j=2}^{r} \pi_j C_j \tag{15}$$

where N_j and C_j are defined in section 4.

5.2 Case of k Absorbing States

Let the Markov Chain $\{X_t\}$ has states $S = \{1,2,...,k,k+1,...,r\}; k < r$ with transition matrix P. Assume $\{X_t\}$ has k absorbing states and (r-k) transition states denoted by $R = \{1,2,...,k\}$ and $T = \{k+1,...,r\}$ respectively, i.e.

$$P = \begin{bmatrix} I & 0 \\ P_1 & Q \end{bmatrix} \tag{16}$$

where I is the $k \times k$ identity matrix, Q is $(r-1)\times(r-1)$ irreducible and aperiodic matrix of transmission probabilities $P_{ij}; i, j \in T$, P_1 is $(r-k)\times k$ and 0 is $k \times (r-k)$ zero matrix.

Now, assuming $\rho_1, \rho_2, ..., \rho_n$ are eigenvalues of Q such that $|\rho_1| > |\rho_2| \geq |\rho_3| \geq .... \geq |\rho_n|$; $\rho_1 < 1$ and the right and the left eigenvalues are $\{a_j\}$ and $\{b_j\}$, $j \in T$ respectively defined in section 5.1. Forming a new Markov Chain by conditioning an eventual absorption into a particular fixed state, say {1}. In this case, we obtain a new absorbing chain $\{X_t^*\}$ with single absorbing state {1}. The transition states will remain as before, but will be governed by a new transition matrix P*; i.e.

$$P^* = \begin{bmatrix} 1 & 0 \\ P_1^* & Q^* \end{bmatrix}; \qquad P_1^* \neq 0 \tag{17}$$

Using the same approach as in section 5.1 and Al-Towaiq and Al-Eideh [8], it is easily shown that

$$\lim_{n\to\infty} P(X_n^* = j \mid X_n^* \in T) = \frac{B_{j1} b_j}{\sum_{j\in T} B_{j1} b_j} \tag{18}$$

where $B_{j\ell}$ is the probability that the process starting in transient state j ends up in absorbing state ℓ ; $\ell = 1,2,...,k$ and can be calculated by

$$B_{j\ell} = P_{j\ell} + \sum_{h\in T} P_{jh} B_{h\ell}; \quad j \in T \,\&\, \ell \in R \tag{19}$$

Also,

$$\lim_{n\to\infty}\lim_{m\to\infty} P(X_m^* = j \mid X_n^* \in T) = b_j a_j \quad ; j \in T \,.$$

This means that the steady state distribution π_j for the amount of electricity enters some of GCC countries say k and no amount exit by conditioning one country is given by

$$\pi_j = a_j b_j \tag{20}$$

Note that this result does not change and remains the same as in section 5.1. Therefore, the long run average of electricity (LRAOE) enters a *k* countries conditional on the eventual absorption into one country is given by:

$$\text{LRAOE} = \sum_{j=k+1}^{r} \pi_j N_j \tag{21}$$

and the associated long run mean cost per unit of time is also given by

$$\text{LRMC} = \sum_{j=k+1}^{r} \pi_j C_j \tag{22}$$

where C_j is defined in section 4.

6. Applications

Example 6.1

Consider a group of 4 countries which means we have 4 inputs and 4 outputs. Suppose the observed amount of electricity $N_{ij}; i, j = 1,2,3,4$ entering approach i and exiting approach j in a period of time are given as in the following table:

$$N = \begin{bmatrix} 50 & 0 & 0 & 0 \\ 30 & 38 & 40 & 28 \\ 31 & 41 & 45 & 30 \\ 22 & 29 & 42 & 40 \end{bmatrix} \qquad \begin{bmatrix} \frac{N_i}{50} \\ 136 \\ 147 \\ 133 \end{bmatrix}$$

Here each row represents an entrance state and each column represents an exit state. The maximum likelihood estimate of P_{ij} is given by

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.22 & 0.28 & 0.3 & 0.2 \\ 0.21 & 0.28 & 0.31 & 0.2 \\ 0.16 & 0.22 & 0.32 & 0.2 \end{bmatrix}.$$

For example, the proportion of amount of electricity that come from a country (entrance) 2 and leaves through country (exit) 3 is 0.3. Note that state

Let

$$P_o = \begin{bmatrix} 0.22 \\ 0.21 \\ 0.16 \end{bmatrix} \ \& \ Q = \begin{bmatrix} 0.28 & 0.3 & 0.2 \\ 0.28 & 0.31 & 0.2 \\ 0.22 & 0.32 & 0.3 \end{bmatrix}.$$

Now, solving $|Q - \lambda I| = 0$ we get the eigenvalues $\lambda_1 = 0, \lambda_2 = 0.6, \lambda_3 = 0.84.$ Then the maximum modulus eigenvalue $\rho_1 = 0.84$. The associated normalized right and left eigenvalues $\underline{a}$ and $\underline{b}$ of Q respectively corresponding to $\rho_1 \left(\sum a_j b_j = 1 \ \& \ \sum b_j = 1 \right)$ and given by:

$$\underline{a} = [0.928 \quad 1.1 \quad 0.9687]$$

and

$$\underline{b} = \begin{bmatrix} 0.3 \\ 0.31 \\ 0.39 \end{bmatrix}$$

Thus, using Eq.(13), we suggest the steady state distribution $\pi_j; j = 2,3,4$ as

$$\underline{\pi} = (0.2786,\ 0.3436, 0.3778).$$

This means that in the long run the proportion of amount of electricity that go through exit 2 is 0.2786.

Now, since N_2 = 136, N_3 = 147, N_4 = 133, the long run average of amount of electricity per unit of time as in Eq(14) is given by:

$$LRAOE = 138.65$$

i.e. approximately 139 kW of electricity per unit of time. This means that almost 46.33 Watts of electricity will go through each exit 2, 3, and 4 per unit of time.

Also, assume the cost of Watts per kilo incurred by each exit j = 2, 3, 4 is given by C_2 = 18 cents, C_3 = 15 cents, C_4 = 17 cents. Then, the long run mean cost of electricity using Eq.(15) is given by

$$LRMC = 16.59$$

i.e almost 17 cents, the long run mean cost of electricity.

Example 6.2

Consider a group of 6 countries. Similarly as in example 6.1 above, the observed amount of electricity $N_{ij}; i, j = 1,2,..,6$ is given by:

$$N = \begin{bmatrix} 43 & 0 & 0 & 0 & 0 & 0 \\ 0 & 60 & 0 & 0 & 0 & 0 \\ 0 & 0 & 50 & 0 & 0 & 0 \\ 22 & 40 & 30 & 38 & 40 & 28 \\ 38 & 32 & 31 & 41 & 45 & 30 \\ 20 & 18 & 22 & 29 & 42 & 40 \end{bmatrix} \quad \begin{bmatrix} N_i \\ 43 \\ 60 \\ 50 \\ 198 \\ 217 \\ 171 \end{bmatrix}$$

and the maximum likelihood estimate of $P_{i,j}$ is given by

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0.11 & 0.2 & 0.15 & 0.19 & 0.2 & 0.14 \\ 0.18 & 0.15 & 0.14 & 0.19 & 0.21 & 0.14 \\ 0.12 & 0.11 & 0.13 & 0.17 & 0.25 & 0.14 \end{bmatrix}$$

Here,

$$P_o = \begin{bmatrix} 0.11 & 0.2 & 0.15 \\ 0.18 & 0.15 & 0.14 \\ 0.12 & 0.11 & 0.13 \end{bmatrix} \; \& \; Q = \begin{bmatrix} 0.19 & 0.2 & 0.14 \\ 0.19 & 0.21 & 0.14 \\ 0.17 & 0.25 & 0.23 \end{bmatrix}$$

Using the same analysis we get the maximum modulus eigenvalue $\rho_1 = 0.525$ and the normalized right and left eigenvalues $\underline{a}$ and $\underline{b}$ corresponding to ρ is given by:

$$\underline{a} = \begin{bmatrix} 0.78 & 1.251 & 0.962 \end{bmatrix}$$

and

$$\underline{b} = \begin{bmatrix} 0.338 \\ 0.345 \\ 0.317 \end{bmatrix}$$

Now, using Eq.(20), the steady state distribution $\pi_j; j = 4,5,6$ is

$$\pi = (0.263, \quad 0.432, \ 0.305)$$

Since N_4 = 198, N_5 = 217, N_6 = 171, the long run average of amount of electricity per unit of time as in Eq(21) is given by:

$$LRAOE = 197.973 \sim 198$$

This means that almost 66 kW of electricity will go through each exit 4, 5, and 6 per unit of time. Finally, if we assume that the cost as before in example 6.1 above:

C_4 = 18 cents, C_5 = 15 cents, C_6 = 17 cents.

Then, the long run mean cost of electricity using Eq.(122) is given by

$$LRMC = 16.399$$

Which means approximately 16.4 cents, the long run mean cost of kilos of Watts per unit of time.

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