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Economic Order Quantity for Items with Two Types of Imperfect Quality

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Abstract. The classical economic order quantity (EOQ) considers that the ordered items are of perfect quality. In this research, a model for the economic order quantity of imperfect quality items is developed, where the incoming lot has fractions of scrap and re-workable items. These fractions are considered to be random variables with known probability density functions. The demand is satisfied from perfect items and reworked items; whereas the scrap items are sold in a single batch at the end of the cycle with a salvage cost. A numerical analysis is provided to illustrate the sensitivity of the model to the fractions of scrap and reworked items.

Keywords: Inventory, Imperfect quality, Economic order quantity, Inspection rate, Rework rate.
AMS Classification: 90B05

Notation

y : Order size
D : Demand rate
c : Unit variable cost
k : Fixed cost of placing an order
P_s : Percentage of scrap items
P_R : Percentage of re-workable items
P : Percentage of scrap and re-workable items
$f(P_s)$: Probability density function of P_s
$f(P_R)$: Probability density function of P_R
t_1 : Inspection period
t_2 : Rework period
t_3 : Remaining period to consume the entire inventory after receiving reworked items.
Z_1 : Inventory level after the inspection period
Z_2 : Inventory level after the selling of the scrap items and return reworked items
Z_3 : Inventory level just before receiving the reworked items
Z_4 : Inventory level just after receiving the reworked items
a : Unit selling price of items of good quality
b : Unit selling price of scrap items, $v < c$.
x : Inspection rate
L : Rework rate
h : Unit holding cost
d : Unit inspection cost
R : Unit rework cost
T : Cycle length

1. Introduction

Although the economic order quantity is an old topic for research, it still is of wide interest to many industries today. This is because of the weakness of the developed economic order models that are based on unrealistic assumptions. This has led to the search for more realistic EOQ models which represent actual and real life cases. The main assumption in classical order quantity is that all items received are perfect; this particular assumption has been challenged by researchers.

Lee and Rosenblatt [1] have studied the effects of an imperfect production process on the optimal production cycle time. Cheng [2] has proposed an

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EOQ model with a demand dependent unit production cost and imperfect production processes. They formulate their inventory decision problem as a geometric program and solve the model to obtain closed form optimal solutions. Khouja [3] has extended the economic production quantity model to cases where production rate is a decision variable so that the unit production cost is a function of the production rate. Chang [4] has investigated the inventory problem for items received with imperfect quality, where, upon the arrival of the ordered lot, 100% inspection process is performed and the items of imperfect quality are sold as a single batch at a discounted price, prior to receiving the next shipment. Chiu et al. [5] have considered the economic production quantity model with a random defective rate and an imperfect rework process. Rezaei [6] has extended the traditional EPQ/EOQ model by accounting for imperfect quality items when using the backorder EPQ/EOQ formulae. An integrated EPQ model with imperfect quality and machine failure is proposed by Das et al. [7]. Yassine et al. [8] have developed an EPQ model with disaggregation and consolidation of imperfect quality shipments. Al-Salamah [9] has developed an EOQ model when a single acceptance sampling plan with destructive testing and inspection errors is adopted. It is assumed that when the lot is rejected, items in the rejected lot are sold at a secondary market at a reduced price and the buyer will place another order.

Eroglu and Ozdemir [10] have developed an EOQ model in which each incoming lot contains some defective items and shortages are backordered. They assume that each lot goes through a 100% inspection to separate good from defective items; the defective items are classified as imperfect quality and scrap items. At the end of screening process, imperfect-quality items are sold as a single lot and scrap items are disposed of from inventory with disposal cost. Wee et al. [11] have developed an optimal inventory model for items with imperfect quality and shortage backordered, where poor-quality items exist during production. Poor quality items are picked up during the screening process and are withdrawn from stock instantaneously. Chen et al. [12] have proposed a fuzzy economic production quantity model with imperfect products that can be sold at a discount price. Yoo et al. [13] have proposed a profit-maximization economic production quantity model that incorporates both imperfect production quality and two-way imperfect inspection. Lin [14] has developed an inventory model for items with imperfect quality and quantity discounts where buyer has exerted power over its supplier. Ho et al [15] integrate the vendor and the buyer in a model, in which, the order lot contains a random proportion of defective items and partial backlogging is allowed.

Salameh and Jaber [16] have hypothesized a production/inventory situation where items, received or produced, are not of perfect quality. They consider the issue that poor quality items are sold as a single batch by the end of the 100% inspection process. Hayek and Salameh [17] have studied the effect of imperfect quality items on the finite production model. When production stops, defective items are assumed to be reworked at a constant rate. And the optimal operating policy that minimizes the total inventory cost per unit time for the finite production model under the effect of imperfect quality is derived where shortages are allowed and backordered. Papachristos and Konstantaras [18] have examined models without shortages, probabilistic proportional imperfect quality, and withdrawing at the end of the planning horizon. Maddah and Jaber [19] have rectified a flaw in an economic order quantity model with unreliable supply, characterized by a random fraction of imperfect quality items and a screening process, developed by Salameh and Jaber [16]. Then, they have analyzed the effect of screening speed and variability of the supply process on the order quantity. In addition, they extend the model by allowing for several batches of imperfect quality items to be consolidated and shipped in one lot. Jaber et al. [20] have extended the work of Salameh and Jaber [16] by assuming the percentage defective per lot reduces according to a learning curve. Khan et al. [21] have extended Salameh and Jaber's work for the case where there is learning in inspection. They consider situations of lost sales and backorders. Khan et al. [22] have used approach similar to Salameh and Jaber [16] to produce an optimal production/order quantity that takes care of imperfect processes, where the inspector may commit errors while screening.

In this article, we develop an inventory model where the incoming lot has imperfect quality items, either scrap or re-workable. When the lot of size y is received, it is subjected to 100% inspection with a constant inspection rate. Inspection takes time t_1. After screening, items will be classified in one of the following types: scrap, re-workable, or good. We assume that the probability density functions of the fractions of the scrap and re-workable items (P_s and P_R, respectively) are known. The defective items are sold in a secondary market with a salvage cost, and re-workable items are returned to the supplier to be reworked and received back as good within the cycle of inspection. We assume the rework process is error free.

The article is organized as follows: Section 2 describes the model. Section 3 develops the mathematical model. Section 4 presents the numerical example. Section 5 discusses the sensitivity analysis. The article is concluded by Section 6.

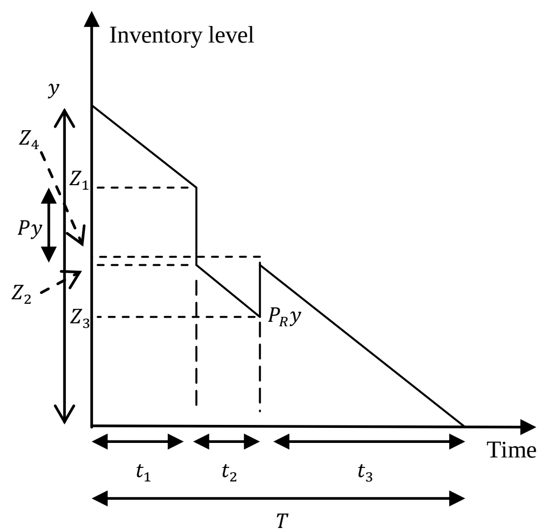

Figure 1. Behavior of inventory level over time

2. Model Description

Figure 1 represents the model where the lot of size y is received with purchasing price of c per unit and the ordering cost of k. It is assumed that each order contains a fraction of scrap and re-workable items P_s and P_R with known probability density functions $f(P_s)$ and $f(P_R)$, respectively. Good and reworked items can be sold at a per unit. On the other hand, scrap items will be sold in a batch at the end of the cycle with salvage (discount) v per unit. The optimal order quantity is found by taking the difference between the total revenue and total cost, the latter of which consists of four types: procurement cost, inspection cost, rework cost, and inventory carrying cost. Revenues come from selling of good items and scrap items. The main assumptions of this model are:

i. Shortages are not allowed.
ii. Inspection and rework processes are error free.
iii. The quantity of good items is sufficient to satisfy the demand during period of inspection.

3. Mathematical Model

Since shortage is not allowed, to avoid shortage, the number of good items is at least equal to the demand during inspection time:

$$(1 - P_s - P_R)y \geq Dt_1 \qquad (1)$$

By substitution of the value of $t_1 = \frac{y}{x}$, we get:

$$P_s + P_R \leq 1 - \frac{D}{x} \qquad (2)$$

where $x \geq D$.

Since P_s and P_R are coming from a probability density functions (p.d.f), they will be limited as the following expression:

$$E(P_s) + E(P_R) \leq 1 - \frac{D}{x}, \text{ for } x \geq D \qquad (3)$$

The time t_1 needed to inspect the lot is:

$$t_1 = \frac{y}{x} \tag{4}$$

The inventory level just before the end of the inspection process is:

$$Z_1 = y - \frac{Dy}{x} = \left(1 - \frac{D}{x}\right) y \tag{5}$$

The time t_2 (rework time) between the time the re-workable items are sent and the time they are received back is:

$$t_2 = \frac{E(P_R)y}{L} \tag{6}$$

Inventory level after the removal of the scrap items and the return of the reworked items is:

$$Z_2 = \left(1 - \frac{D}{x}\right) y - Py = \left(1 - P - \frac{D}{x}\right) y \tag{7}$$

Inventory level just before receiving the reworked items is:

$$Z_3 = \left(1 - P - \frac{D}{x} - \frac{D \cdot E(P_R)}{L}\right) y \tag{8}$$

Inventory level when the reworked items are put in the inventory is:

$$Z_4 = \left(1 - P_s - \frac{D}{x} - \frac{D \cdot E(P_R)}{L}\right) y \tag{9}$$

Finally, the time to consume Z_4 is:

$$t_2 = \frac{Z_4}{D} \tag{10}$$

The expected total revenue is the summation of sales of the good items and scrap items and it is given as:

$$E\big(TR(y)\big) = a(1 - E(P_s))y + b \cdot y \cdot E(P_s) \tag{11}$$

The expected total comprises four different costs. The first cost is the procurement cost:

$$PC(y) = k + cy \tag{12}$$

The rework cost is:

$$RC(y) = R \cdot E(P_R) y \tag{13}$$

The inspection cost is:

$$IC(y) = dy \tag{14}$$

And the expected inventory holding cost is:

$$E\big(HC(y)\big) = h\left[\frac{E(1-P_s)yE(T)}{2} + E(P_s)yt_1 - E(P_R)yt_2\right] = h\left[\frac{E(1-P_s)yE(T)}{2} + \frac{E(P_s)y^2}{x} - \frac{E(P_R^2)y^2}{L}\right] \tag{15}$$

The expression for the expected total cost is:

$$E\big(TC(y)\big) = k + cy + R \cdot E(P_R)y + dy + h\left[\frac{E(1-P_s)yE(T)}{2} + \frac{E(P_s)y^2}{x} - \frac{E(P_R^2)y^2}{L}\right] \tag{16}$$

The expected total profit equals the expected total revenues minus the expected total cost:

$$E\big(TP(y)\big) = E\big(TR(y)\big) - E(TC(y)) = a(1 - E(P_s))y + b \cdot y \cdot E(P_s) - \left[k + cy + R \cdot E(P_R)y + dy + h\left[\frac{E(1-P_s)yE(T)}{2} + \frac{E(P_s)y^2}{x} - \frac{E(P_R^2)y^2}{L}\right]\right] \tag{17}$$

The expected cycle period is given by:

$$E(T) = \frac{E(1-P_s)y}{D} \tag{18}$$

The expected total profit per unit time is:

$$E(TPU(y)) = \frac{E(TP(y))}{E(T)} = \left\{\left[a(1-E(P_s)) + bE(P_s) - c - RE(P_R) - d\right]D - \frac{kD}{y} - hy\left[\frac{E((1-P_s)^2)}{2} + \frac{E(P_s)D}{x} - \frac{E(P_R^2)D}{L}\right]\right\}/\{1-E(P_s)\} \quad (19)$$

To find the optimal order quantity, the first derivative of Eq. (19) is taken, set to zero, and solved for y:

$$\frac{\partial E(TPU(y))}{\partial y} = \frac{\frac{kD}{y^2} - h\left[\frac{E((1-P_s)^2)}{2} + \frac{E(P_s)D}{x} - \frac{E(P_R^2)D}{L}\right]}{1-E(P_s)} = 0 \quad (20)$$

From Eq. (20), we find the expression of the economic order quantity EOQ_2:

$$EOQ_2 = \left(2kD/\left\{h\left[E((1-P_s)^2) + \frac{2DE(P_s)}{x} - \frac{2DE(P_R{}^2)}{L}\right]\right\}\right)^{\frac{1}{2}} \quad (21)$$

The second derivative is equal to:

$$\frac{\partial^2 E(TPU(y))}{\partial^2 y} = -\frac{2kD}{y^2(1-E(P_s))} \quad (22)$$

Since the second derivative is always negative under all values of y, this means that there exists a unique value of y^* that maximizes Eq. (19).

Note that when $P_S = 0$ and $P_R = 0$, Eq. (21) reduces to the classical formula of EOQ_1:

$$EOQ_1 = \sqrt{\frac{2kD}{h}} \quad (23)$$

4. Numerical Example

From the expression of the expected total profit, there are several terms in $E(TPU(y))$ that are not functions of y. Therefore, these terms will be dropped from Eq. (21); and the new objective function will be in terms of minimizing the expected (relevant) cost per unit time:

$$EC(y) = \left\{\frac{kD}{y} + hy\left[\frac{E((1-P_s)^2)}{2} + \frac{DE(P_s)}{x} - \frac{DE(P_R{}^2)}{L}\right]\right\} /\{1-E(P_s)\} \quad (24)$$

The values in the numerical example are:

Demand rate, D = 50,000 units/year

Unit variable cost, c = \$25/unit

Order fixed cost, k = 100/order

Inventory holding cost, h = \$5/unit/year

Inspection rate, x = 1 unit/min

Unit inspection cost, d = \$0.5/unit

Rework rate, L = 0.5 unit/min

Unit rework cost, R = \$2.5/unit

Unit selling price of good quality items, a = \$50/unit

Unit selling price of scrap items, b = \$20/unit.

We assume the operation of the inventory model operates 8 hours a day, for 365 days a year, so the annual inspection rate is

$$1\ unit/\text{min} \times \left(\frac{60*8*365\ min}{year}\right) = 175{,}200\ unit/year.$$

Also, we assume scrap and re-workable fractions, P_s and P_R, are uniformly distributed with p.d.f as the following:

$$f(P_s) = \begin{cases} 4, & 0 \le P_s \le 0.25 \\ 0 & otherwise \end{cases} \quad (25)$$

$$f(P_R) = \begin{cases} 12.5, & 0 \le P_s \le .08 \\ 0 & otherwise \end{cases} \quad (26)$$

The expected values in the model when the (p.d.f) is a uniform distribution are:

$$E(p) = \frac{1}{b-a}\int_a^b p\,dp = \frac{b^2}{2(b-a)} - \frac{a^2}{2(b-a)}$$

$$E(p^2) = \frac{1}{b-a}\int_a^b p^2 dp = \frac{b^3}{3(b-a)} - \frac{a^3}{3(b-a)}$$

$$E((1-p)^2) = \frac{1}{b-a}\int_a^b (1-p)^2 dp = \frac{\mathrm{a}^2 + \mathrm{ab} + \mathrm{b}^2}{3} + 1 - a - b$$

Since probabilities always positive and $a = 0$:

$$E(p) = \frac{b}{2}$$

$$E(p^2) = \frac{b^2}{3}$$

$$E((1-p)^2) = \frac{\mathrm{b}^2}{3} + 1 - \mathrm{b}$$

$$E(P_s) = 0.125, \quad E(P_R{}^2) = 0.002133$$

$$E((1-P_s)^2) = 0.770833.$$

Then, the optimal value of y that optimizes the expected total profit per unit time is given by:

$$EOQ_2 = \sqrt{\frac{2*100*50000}{5\left[0.770833 + \frac{2*50000*0.125}{175200} - \frac{2*50000*0.002133}{43800}\right]}}$$

$= 1{,}537$ units, with optimal expected cost per unit time $EC(y)$ = $ 7,395

From the classical EOQ model, the optimal value of y, which is given by Eq. (23), is:

EOQ_1 = 1,414 Units, with expected cost per unit time $EC(y)$ = $ 7,424

5. Sensitivity Analysis:

In this section, we will study the effect of the variation in the fractions of the scrap and re-workable items on EOQ_2. We assume that the scrap and re-workable fractions are uniformly distributed with P_s ~U (0, b_1) and P_R ~U (0, b_2), respectively.

The results are generated and tabulated as follows:

1. EOQ_2 values with different fractions of scrap and re-workable items are shown in Table 1.
2. The ratio of EOQ_2 to EOQ_1 with different fractions of scrap and re-workable items are shown in Table 2.
3. The expected cost using EOQ_2 with different fractions of scrap and re-workable items are shown in Table 3.
4. The expected cost using EOQ_1 with different fractions of scrap and re-workable items are shown in Table 4.
5. The penalty for the deviation from EOQ_2, calculated by $\frac{|\mathrm{EC(EOQ_2)} - \mathrm{EC(EOQ_1)}|}{\mathrm{EC(EOQ_2)}}$, with different fractions of scrap and re-workable items are shown in Table 5.

From Table 1, it is shown that $\mathrm{EOQ_2}$ increases with the variation in P_s; for instance, if we fix $b_2 = 0.04$, then when $b_1 = 0.08$, $\mathrm{EOQ_2} = 1{,}456$ units; and when $b_1 = 0.2$, $\mathrm{EOQ_2} = 1{,}517$ unit. Also, $\mathrm{EOQ_2}$ increases with the variation in P_R. For $b_1 = 0.08$, the optimal pairs are ($b_2 = 0.04, \mathrm{EOQ_2} = 1{,}456$ units) and ($b_2 = 0.16, \mathrm{EOQ_2} = 1{,}470$ units).

And from Table 3, it is shown that $\mathrm{EC(EOQ_2)}$ increases with the variation in P_s; for instance, if we fix $b_2 = 0.04$, then when $b_1 = 0.08$, $\mathrm{EC(EOQ_2)} = \$7{,}156$; and when $b_1 = 0.2$, $\mathrm{EC(EOQ_2)} = \$7{,}325$. On the other hand, $\mathrm{EC(EOQ_2)}$ decreases with the variation in P_R. For $b_1 = 0.08$, the optimal pairs are ($b_2 = 0.04, \mathrm{EC(EOQ_2)} = \$7{,}156$) and ($b_2 = 0.16, \mathrm{EC(EOQ_2)} = \$7{,}086$).

It's also shown that the developed economic order quantity ($\mathrm{EOQ_2}$) is greater than the classical economic order quality ($\mathrm{EOQ_1}$) under the same variation in P_s and P_R, which is illustrated in Table 2 by the ratio being greater than one. Finally, the expected cost for $\mathrm{EOQ_2}$ is always less than that for $\mathrm{EOQ_1}$ under the same variation in P_s and P_R as shown in Tables 3 and 4.

Table 1. EOQ_2 with different variation of scrap and re-workable items.

		b_2	0.04	0.08	0.12	0.16	0.2
b_1	$E((1-P_s)^2)$		$E(P_R{}^2)$				
		$E(P_s)$	0.00053	0.00213	0.0048	0.00853	0.01333
0.08	0.92213	0.04	1456	1459	1463	1470	1479
0.2	0.81333	0.1	1517	1520	1525	1533	1543
0.32	0.71413	0.16	1577	1581	1587	1595	1606
0.44	0.62453	0.22	1634	1638	1645	1655	1667
0.56	0.54453	0.28	1687	1691	1698	1709	1723

Table 2. The ratio of EOQ_2 to EOQ_1 with different variation for scrap and re-workable items.

		b_2	0.04	0.08	0.12	0.16	0.2
b_1	$E((1-P_s)^2)$		$E(P_R{}^2)$				
		$E(P_s)$	0.00053	0.00213	0.0048	0.00853	0.01333
0.08	0.92213	0.04	1.03	1.03	1.03	1.04	1.05
0.2	0.81333	0.1	1.07	1.08	1.08	1.08	1.09
0.32	0.71413	0.16	1.12	1.12	1.12	1.13	1.14
0.44	0.62453	0.22	1.16	1.16	1.16	1.17	1.18
0.56	0.54453	0.28	1.19	1.20	1.20	1.21	1.22

Table 3. Expected cost using EOQ_2 with different variation for scrap and re-workable items.

		b_2	0.04	0.08	0.12	0.16	0.2
b_1	$E((1-P_s)^2)$		$E(P_R{}^2)$				
		$E(P_s)$	0.00053	0.00213	0.0048	0.00853	0.01333
0.08	0.92213	0.04	7156	7142	7118	7086	7044
0.2	0.81333	0.1	7325	7309	7284	7248	7201
0.32	0.71413	0.16	7549	7532	7503	7463	7411
0.44	0.62453	0.22	7845	7826	7794	7749	7691
0.56	0.54453	0.28	8235	8214	8178	8127	8062

Table 4. Expected cost using EOQ_1 with different variation for scrap and re-workable items.

		b_2	0.04	0.08	0.12	0.16	0.2
			$E(P_R{}^2)$				
b_1	$E((1-P_s)^2)$	$E(P_s)$	0.00053	0.00213	0.0048	0.00853	0.01333
0.08	0.92213	0.04	7159	7145	7123	7091	7051
0.2	0.81333	0.1	7343	7329	7305	7271	7228
0.32	0.71413	0.16	7594	7579	7553	7517	7471
0.44	0.62453	0.22	7927	7911	7883	7845	7795
0.56	0.54453	0.28	8363	8345	8316	8274	8220

Table 5. The penalty for deviation from EOQ_2 with different variation for scrap and re-workable items

		b_2	0.04	0.08	0.12	0.16	0.2
			$E(P_R{}^2)$				
b_1	$E((1-P_s)^2)$	$E(P_s)$	0.00053	0.00213	0.0048	0.00853	0.01333
0.08	0.92213	0.04	0.04	0.05	0.06	0.08	0.10
0.2	0.81333	0.1	0.25	0.26	0.29	0.33	0.38
0.32	0.71413	0.16	0.60	0.62	0.66	0.73	0.82
0.44	0.62453	0.22	1.05	1.09	1.15	1.24	1.36
0.56	0.54453	0.28	1.56	1.60	1.68	1.80	1.96

6. Conclusion

The economic order quantity is developed for the inventory model with imperfect quality items. The model proposed here considers that the incoming lot has a fraction of scrap and re-workable items, and the lot will go through a 100% inspection. The scrap items will be sold in a secondary market and re-workable items will be returned back to the supplier and received again within the same cycle as good items.

Furthermore, an expression for the optimal lot size has been developed. The optimal lot size is affected by the fraction of imperfect quality items; the lot size increases with the increase in the fraction of scrap items and re-workable items. Also the expected total profit decreases with the fraction of scrap and re-workable items.

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