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Equity in multiproduct supply chain network

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Abstract. In this paper, multiproduct supply chain network is investigated with equity consideration, namely, obtaining the optimal flow pattern, in such a way that no user in the network can increase his(her) benefit with change in product's sending path. However, each kind of products, has an individual cost function and, at the same time, contributes to its own and other product's cost function in an individual way. An algorithm is developed to find optimal flow pattern for such multiproduct supply chain network.

Keywords: Equity; optimal flow pattern; multiproduct supply chain; antitrust law.
AMS Classification: 90C25

1. Introduction

A supply chain is a network of retailers, distributors, transporters, storage facilities, and suppliers that participate in the production, delivery, and sale of products to the consumer. The supply chain is typically made up of multiple companies who coordinate activities to set themselves apart from the competition.

Today, many sectors, in particular, government sectors employ equity objectives to obey antitrust law. In this paper, equity in supply chain network is analyzed as following the competition law.

The competition law, known in the United States as antitrust law, is a law that promotes or maintains market competition by regulating anti-competitive conduct [9]. The history of competition law reaches back to the Roman Empire. The business practices of market traders, guilds and governments have always been subject to scrutiny, and sometimes severe sanctions. Since the 20th century, competition law has become global. The two largest and most influential systems of competition regulation are United States antitrust law and European Union competition law. National and regional competition authorities across the world have formed international support and enforcement networks.

Competition law, or antitrust law, has three main elements:

1) prohibiting agreements or practices that restrict free trading and competition between business. This includes in particular the repression of free trade caused by cartels.

2) banning abusive behavior by a firm dominating a market, or anti-competitive practices that tend to lead to such a dominant position. Practices controlled in this way may include predatory pricing, tying, price gouging, refusal to deal, and many others.

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3) supervising the mergers and acquisitions of large corporations, including some joint ventures. Transactions that are considered to threaten the competitive process can be prohibited altogether, or approved subject to remedies such as an obligation to divest part of the merged business or to offer licenses or access to facilities to enable other businesses to continue competing.

Substance and practice of competition law varies from jurisdiction to jurisdiction. Protecting the interests of consumers (consumer welfare) and ensuring that entrepreneurs have an opportunity to compete in the market economy are often treated as important objectives. Competition law is closely connected with law on deregulation of access to markets, state aids and subsidies, the privatization of state owned assets and the establishment of independent sector regulators, among other market-oriented supply-side policies. In recent decades, competition law has been viewed as a way to provide better public services. Robert Bork [2] has argued that competition laws can produce adverse effects when they reduce competition by protecting inefficient competitors and when costs of legal intervention are greater than benefits for the consumers.

Today, supply chains are more extended and complex than ever before. At the same time, the current competitive economic environment requires that firms operate efficiently, which has spurred interest among researchers as well as practitioner to determine how to utilize supply chains more effectively and efficiently.

Furthermore, although there are numerous articles discussing multi-echelon supply chains, the majority deal with a homogeneous product, but, in application, deal is with multiproduct supply chain. Therefore, homogenous restriction is relaxed and worked with multiproduct in same supply chain network. Note that, in this paper, deal is not with setup cost of network but it is with transfer cost of products.

In this paper, an algorithm is suggested to construct an optimal flow pattern which obeys the well known antitrust law for preventing the users from increasing their benefits or monopolizing any part of trade among the several parts of the network.

Note that Min and Zhou [6] provided a synopsis of supply chain modeling and the importance of planning, designing, and controlling the supply chain as a whole.

2. Supply Chain Network Structure

Assume that multiproduct supply chain network involves two firms A, B, as depicted in Figure 1. Let $G_i = [N_i, L_i]; i = A, B$ denote the graphs consisting of nodes $[N_i]$ and directed links $[L_i]$. Firm $i; i = A, B$ is involved in the production, storage, and distribution of J products, denote typical product by superscript j.

Assume that, firm $i; i = A, B$, has m facilities manufacturing, $M_1^i, \dots, M_m^i$ and n distribution centers that are denoted by $D_1^i, \dots, D_n^i$ without storage and s distribution center with storages are denoted by $S_1^i, \dots, S_s^i$ and has r retail outlets that are denoted by $R_1^i, \dots, R_r^i$.

A path consists of a sequence of links originating at a node $i; i = A, B$ and denotes supply chain activities comprising manufacturing, storage, and distribution of the products to the retail nodes.

Assume node $i; i = A, B$, be the origin, R_k^i, $k = 1, \dots, r$ be the destinations and every origin-destination pair, O/D, be denoted by w.

Let x_p^j denote the nonnegative flow of product j, on path p, f_a^j, flow of product j on link a and let p_w, denote the set of paths connecting the origin/destination pair of node w. Also, let P, denote the set of all paths in the network and W, denote the set of all O/D pairs of nodes. The path flows are grouped into the vector x and link flows into the vector f. In other words, the following notations are used:

$$x \equiv \{x_p : p \in P\}, \quad x_p \equiv (x_p^1, \dots, x_p^J),$$
$$f \equiv \{f_a : a \in L\}, \quad f_a \equiv (f_a^1, \dots, f_a^J).$$

Assume that, d_w^j, denotes the demand for product j; $j = 1, \dots, J$ and O/D pair $w \in W$, along with

$$D \equiv \{d_w : w \in W\}, \quad d_w \equiv (d_w^1, \dots, d_w^J).$$

The links from the top-tiered i, to the manufacturing nodes $M_1^i, \dots, M_m^i$, in Figure 1, represent the manufacturing links. The links from the manufacturing nodes, in turn, to the distribution center nodes, correspond to the shipment links. The links joining first distribution center nodes and second distribution center nodes, correspond to the storage links for the products. Finally, the links joining second distribution center nodes to retails, correspond to the storage links for the products.

In this paper, supply chain model is constructed to include several products in the same supply chain network and flow of each products effects on another products such that each of which has an individual cost function and, at the same time, contributes also to the cost function of the other products.

Let, $c_a^j(f_a^1, \dots, f_a^J)$, denote the cost of one unit shipment of product j, $j = 1, \dots, J$ on link a, which is a function of other product's flows on same link. That is:

$$c_a^j = c_a^j(f_a^1, \dots, f_a^J), \quad j = 1, \dots, J, \quad \forall a \in L. \tag{1}$$

In other words,

$$C \equiv \{c_a : a \in L\}, \quad c_a \equiv (c_a^1, \dots, c_a^J).$$

Assume that there is complete interaction among all products of typical j on a link a, the share of link cost to each of these products will be given by

$$\bar{c}_a^j = \bar{c}_a^j(f_a^1, \dots, f_a^J) = \frac{c_a^j(f_a^1, \dots, f_a^J)}{f_a^j}. \tag{2}$$

Therefore, the personal cost $\bar{C}_p^j$ to a product of typical j who sent on path p will be given by

$$\bar{C}_p^j = \sum_{a \in L} \bar{c}_a^j(f_a^1, \dots, f_a^J)\delta_{ap}, \quad \forall p \in P, \tag{3}$$

where $\delta_{ap} = 1$, if link a is contained in path P and $\delta_{ap} = 0$, otherwise.

For example, assume that a personal cost function for a product j provided by the quadratic model with the following property:

$$\begin{aligned} &\bar{c}_a^j(f_a^1, \dots, f_a^J) = \textstyle\sum_{l=1}^J g_a^{jl} f_a^j f_a^l + h_a^j f_a^j, \\ &\forall a \in L, \quad j = 1, \dots, J, \end{aligned} \tag{4}$$

where g_a^{jl} , h_a^j are given constants. This example is investigated in details in section 5

The triple $T = (G, D, C)$ will be called a supply chain network, where G is a directed network, D is a demand vector and C is a cost vector as defined before.

The following conservation of flow equations must hold for firm $i; i = A, B$ in which, each product j, and each O/D pair, w:

$$\sum_{p \in P_w} x_p^j = d_w^j, \quad j = 1, \dots, J, \quad \forall w \in W, \tag{5}$$

that is, the demand for each product must satisfy each retail outlet.

A flow pattern x, with the demand D, is called feasible, if equation (5) satisfies for every $w \in W$.

Every flow pattern x, generates a load pattern f and x is called compatible with f.

A flow pattern f, is called feasible, if there exists at least one feasible flow pattern x, compatible with f and the following flow equation holds:

$$f_a^j = \sum_{p \in P} x_p^j \delta_{ap}, \quad j = 1, \dots, J, \quad \forall a \in L, \tag{6}$$

where $\delta_{ap} = 1$, if link a is contained in path P and $\delta_{ap} = 0$, otherwise.

Expression (6) states that the flow on a link a is equal to the sum of all the path flows on path p containing link a.

The path flows must be nonnegative, that is:

$$x_p^j \geq 0, \quad j = 1, \dots, J, \quad \forall p \in P. \tag{7}$$

In the equity problem one seeks to determine the path flow pattern x^* (and the corresponding link flow pattern f^*) which satisfies the conservation of flow equation (5), (6) and (7) which also satisfies the supply chain network equity conditions given by the following statement. For each pair $w \in W$, each product and each path $p \in P_w$ [7]:

$$\bar{C}_p^j \begin{cases} = \lambda_w^j, & \text{if} \quad x_p^j > 0, \\ \geq \lambda_w^j, & \text{if} \quad x_p^j = 0. \end{cases} \tag{8}$$

Indeed, conditions (8) mean that only those paths connecting an O/D pair for product j will be used which have equal and minimal personal costs. In (8) the minimal cost for O/D pair w and typical product j is denoted by λ_w^j and its value is obtained once the optimal flow pattern is determined. Otherwise, personal cost of a product of the network could be decreased by switching to a path with lower cost.

In the case of personal link cost functions of the form (2), in which the cost on a link is assumed to be continuous and an increasing function of the flow and the symmetry assumption exists, that is, $\frac{\partial \bar{c}_a^j(f_a^1, f_a^2, \dots, f_a^J)}{\partial f_a^l} = \frac{\partial \bar{c}_a^l(f_a^1, f_a^2, \dots, f_a^J)}{\partial f_a^j}$, for all typical products j,l, one can still reformulate the solution to the equity problem in multiproduct supply chain satisfying conditions (8) as the solution to the following optimization problem,

$$C(f^*) = \text{Min} \quad \sum_{j=1}^{J} \sum_{a\in L} \int_0^{f_a^j} \bar{c}_a^j(f_a^1, \ldots, f_a^J) df_a^j \tag{9}$$

s.t. :

$$\sum_{p\in P_w} x_p^j = d_w^j, \quad j = 1, \ldots, J, \ \forall w \in W$$
$$\sum_{p\in P} x_p^j \delta_{ap} = f_a^j, \quad j = 1, \ldots, J, \ \forall a \in L$$
$$x_p^j \geq 0, \quad j = 1, \ldots, J, \quad \forall p \in P$$

The solution to the optimization problem (9) can also be obtained once the optimal flow pattern is determined.

Note that, the K.K.T [1]conditions satisfy for problem (9) which λ_w^j term in (8) corresponds to the lagrange multiplier associated with the constraint (5) for that O/D pair w and typical product j.

The optimal link flow pattern is unique for problem (9), subject to (5)-(7), if the objective function (9) is strictly convex [1].

3. Definition and Theorem

Definition 1. *An optimal flow pattern x for a given multiproduct supply chain network $T = (G, D, C)$ is defined as a feasible flow pattern x that satisfies the following condition: for every fixed typical product j and every pair O/D, $w \in W$, such that $d_w^j > 0$, there exists $\epsilon > 0$ with following property: choose any $p \in P_w$ for which $x_p^j > 0$ and any number Δx in the interval $(0, \min\{\epsilon, x_p^j\}]$, consider another path $q \in P_w$, then the personal cost $\bar{C}_p^j(f)\Delta x$ of Δx products of typical j in the original flow pattern x is no greater than the personal cost $\bar{C}_q^j(\bar{f})\Delta x$ of the same products in the flow pattern $\bar{x}$ defined by*

$$\begin{aligned} &\bar{x}_p^j = x_p^j - \Delta x, \\ &\bar{x}_q^j = x_q^j + \Delta x, \\ &\bar{x}_r^j = x_r^j, \quad r \in P, \quad r \neq q, \quad r \neq p, \\ &\bar{x}_s^l = x_s^l, \quad l \neq j, \quad s \in P. \end{aligned} \tag{10}$$

Theorem 1. *Let f be the (unique) optimal flow pattern. For any $x^0 \in Z$, let $x^n \equiv E^n x^0$ where f^n denotes the flow pattern induce by x^n. Then*

$$f^n \to f \quad as \quad n \to \infty.$$

Proof. it is similar to the proof of theorem 3.1 in Dafermos and Sparrow [4]. □

4. Algorithm

In this section, an algorithm is developed to find an optimal flow pattern for multiproduct supply chain networks which optimized problem (9). The algorithm constructs an optimal flow pattern by iteration, i.e., starting from an arbitrary initial feasible flow pattern[2] x^0, it generates a sequence $\{x^n\}$ of feasible flow patterns converging to the set of optimal flow patterns. The passage from x^{n-1} to x^n is attained by applying an operator E, i.e., $x^n = Ex^{n-1}$. Once E has defined, the description of the algorithm is complete.

Let $Z[T]$ stand for the set of all feasible flow patterns of the supply chain network T. An operator

$$E : Z[T] \to Z[T] \tag{11}$$

is defined as the composition

$$E = E_{w(m)} \circ \cdots \circ E_{w(1)} \tag{12}$$

of operators

$$E_{w(l)} : Z[T] \to Z[T], \quad l = 1, \ldots, m, \tag{13}$$

where $w(1), \ldots, w(m)$ is an arbitrary ordering of the set W. In turn, $E_{w(l)}$ will be defined as the composition

$$E_{w(l)} = E_{w(l)}^J \circ \cdots \circ E_{w(l)}^1 \tag{14}$$

of operators

$$E_{w(l)}^j : Z[T] \to Z[T], \quad j = 1, \ldots, J, \tag{15}$$

where E_w^j sends a feasible flow pattern x to another feasible flow pattern $\hat{x} = E_w^j x$, which is constructed by the following procedure.

Among the elements of P_w, determine the paths q and r requiring

$$\begin{aligned} \bar{C}_q^j(f) &= \min_{p\in P_w} \{\bar{C}_p^j(f)\} \\ \bar{C}_r^j(f) &= \max_{p\in P_w, x_p^j > 0} \{\bar{C}_p^j(f)\} \end{aligned} \tag{16}$$

[1]Karush-Kuhn-Tucker

[2]Such a flow pattern can be obtained with all-nothing assignment method.

where $\bar{C}_p^j(f)$ denotes the personal cost on path p for product j.

If $|\bar{C}_r^j - \bar{C}_q^j| \leq \varepsilon$ with $\varepsilon > 0$, a prespecified tolerance, then stop; otherwise compute δ. In the case of quadratic model (4), δ can be determined explicitly through the formula

$$\delta = \min\{x_r^j, \frac{\bar{C}_r^j(f) - \bar{C}_q^j(f)}{2\sum_{a\in L} g_a^{jj}(\delta_{aq} - \delta_{ar})^2}\}. \tag{17}$$

Then set

$$\begin{aligned} \bar{x}_p^l &= x_p^l \quad l \neq j, \quad p \in P \\ \bar{x}_p^j &= x_p^j \quad p \neq q, \quad p \neq r \\ \bar{x}_q^j &= x_q^j + \delta \\ \bar{x}_r^j &= x_r^j - \delta, \end{aligned} \tag{18}$$

where δ is selected so that the $\bar{x}$ is optimal flow pattern.

5. Numerical Example

In this section, a numerical example is presented for the solution to the supply chain. The example was solved using the algorithm.

In this example, it is assumed that each firm $i; i = A, B$, was involved in the production, storage, and distribution of two products, and each firm had two manufacturing plants, one distribution center, and supplied the products to two retail outlets.

In this example, is assumed that cost function is nonlinear(quadratic), of the form:

$$\bar{c}_a^j(f_a^1, f_a^2) = \sum_{l=1}^2 g_a^{jl} f_a^j f_a^l + h_a^j f_a^j, \quad \forall a \in L, \quad j = 1, 2. \tag{19}$$

The link cost functions for products 1 and 2 were Picked up from [8] that are listed in Table 1.

The demands at the retail outlets for firm A and firm B were set to 5 for each product. Hence, $d_{R_k^i}^j = 5$ for $i = A, B$; $j = 1, 2$, and $k = 1, 2$.

The optimal solution for the product flows for each product is given in Table 2.

Table 2. The resulting optimal flow patterns.

Link a	*FromNode*	*ToNode*	f_a^{1*}	f_a^{2*}
1	A	M_1^A	0.00	0.00
2	A	M_2^A	10.00	10.00
3	M_1^A	D_1^A	0.00	0.00
4	M_2^A	D_1^A	10.00	10.00
5	D_1^A	S_1^A	10.00	10.00
6	D_1^A	R_1^A	5.00	5.00
7	S_1^A	R_2^A	5.00	5.00
8	B	M_1^B	0.00	0.00
9	B	M_2^B	10.00	10.00
10	M_1^B	S_1^B	0.00	0.00
11	M_2^B	D_1^B	10.00	10.00
12	D_1^B	S_1^B	10.00	10.00
13	S_1^B	R_1^B	5.00	5.00
14	S_1^B	R_2^B	5.00	5.00

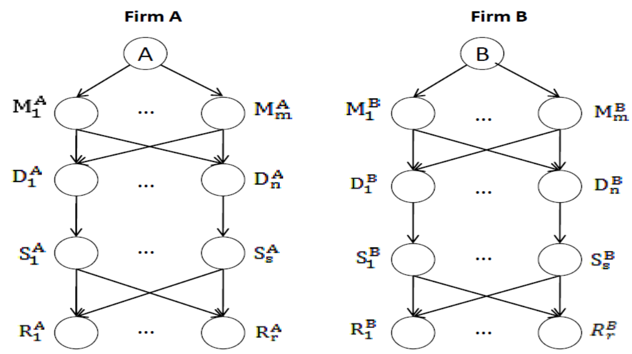

Figure 1. Supply chains of firms A and B.

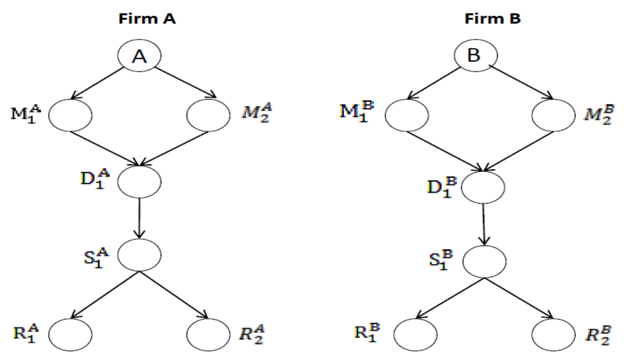

Figure 2. Supply chain network topology for the numerical example.

6. Conclusion

In this paper, equity in multiproduct supply chain network was developed as antitrust law. The optimal flow pattern was constructed for supply chain network utilizing an algorithm. By using this flow pattern, equity was satisfied in supply chain network and antitrust law was satisfied in the network, so that no person or firm in supply chain network can monopolize any parts

Table 1. Definition of links and associated personal link cost functions.

Link a	$FromNode$	$ToNode$	$\hat{c}_a^1(f_a^1, f_a^2)$	$\hat{c}_a^2(f_a^1, f_a^2)$
1	A	M_1^A	$1(f_1^1)^2 + 2f_1^2 f_1^1 + 11f_1^1$	$2(f_1^2)^2 + 2f_1^1 f_1^2 + 8f_1^2$
2	A	M_2^A	$2f_2^1)^2 + 2f_2^2 f_2^1 + 8f_2^1$	$1(f_2^2)^2 + 2f_2^1 f_2^2 + 6f_2^2$
3	M_1^A	D_1^A	$3(f_3^1)^2 + 2.5f_3^2 f_3^1 + 7f_3^1$	$4(f_3^2)^2 + 2.5f_3^1 f_3^2 + 7f_3^2$
4	M_2^A	D_1^A	$4(f_4^1)^2 + 1.5f_4^2 f_4^1 + 3f_4^1$	$3(f_4^2)^2 + 1.5f_4^1 f_4^2 + 11f_4^2$
5	D_1^A	S_1^A	$1(f_5^1)^2 + f_5^2 f_5^1 + 6f_5^1$	$4(f_5^2)^2 + f_5^1 f_5^2 + 11f_5^2$
6	S_1^A	R_1^A	$3(f_6^1)^2 + 1.5f_6^2 f_6^1 + 4f_6^1$	$4(f_6^2)^2 + 1.5f_6^1 f_6^2 + 10f_6^2$
7	S_1^A	R_2^A	$4(f_7^1)^2 + f_7^2 f_7^1 + 7f_7^1$	$2(f_7^2)^2 + 2f_7^1 f_7^2 + 8f_7^2$
8	B	M_1^B	$4(f_8^1)^2 + 3f_8^1 f_8^2 + 5f_8^1$	$4(f_8^2)^2 + 3f_8^1 f_8^2 + 6f_8^2$
9	B	M_2^B	$1(f_9^1)^2 + 1.5f_9^1 f_9^2 + 4f_9^1$	$4(f_9^2)^2 + 1.5f_9^1 f_9^2 + 6f_9^2$
10	M_1^B	D_1^B	$2(f_{10}^1)^2 + 3f_{10}^1 f_{10}^2 + 3.5f_{10}^1$	$3(f_{10}^2)^2 + 3f_{10}^1 f_{10}^2 + 4f_{10}^2$
11	M_2^B	$D_{1,1}^B$	$1(f_{11}^1)^2 + 2.5f_{11}^1 f_{11}^2 + 4f_{11}^1$	$4(f_{11}^2)^2 + 2.5f_{11}^1 f_{11}^2 + 5f_{11}^2$
12	D_1^B	S_1^B	$4(f_{12}^1)^2 + 3f_{12}^1 f_{12}^2 + 6f_{12}^1$	$2(f_{12}^2)^2 + 3f_{12}^1 f_{12}^2 + 5f_{12}^2$
13	S_1^B	R_1^B	$3(f_{13}^1)^2 + 3f_{13}^1 f_{13}^2 + 7f_{13}^1$	$4(f_{13}^2)^2 + 3f_{13}^1 f_{13}^2 + 10f_{13}^2$
14	S_1^B	R_2^B	$4(f_{14}^1)^2 + 0.5f_{14}^1 f_{14}^2 + 4f_{14}^1$	$4(f_{14}^2)^2 + 0.5f_{14}^1 f_{14}^2 + 12f_{14}^2$

of the trade or commerce among the several part of network.

References

[1] Bazaraa, M.S., Sherali, H.D., Shetty, C.M., Nonlinear programming: Theory and algorithms. John Wiley & Sons, New York (2006).

[2] Bork, R.H., The antitrust paradox: A policy at war with itself. Free Press (1993).

[3] Cheng, T.C.E, Wu, Y.N., A multiproduct, multicriterion supply-demand network equilibrium model. Operations Research, 54, 544-554 (2006).

[4] Dafermos, S.C., Sparrow, F.T., An extended traffic assignment model with applications to two way traffic. Transportation Science, 5, 366-389 (1971).

[5] David, D.D., Wilson, B.J., Equilibrium price dispersion, merger and synergies: An experimental investigation of differentiated product competition. International Journal of the Economics of Bussiness, 13, 169-194 (2006).

[6] Min, H., Zhou, G., Supply chain modeling: past, present and future. Computers and Industrial Engineering, 43, 231-249 (2002).

[7] Nagurney, A., Mathematical models of transportation and networks. A Volume in the Encyclopedia of Life Support Systems (EOLSS), United Nations Educational, Scientific and Cultural Organization (UNESCO), Dr. Wei-Bin Zhang, Editor (2007).

[8] Nagurney, A., Woolley, T., Qiang, Q., Multiproduct supply chain horizontal network integration: Models, theory and computational results. International Transactions in Operational Research, 17, 333-349 (2010).

[9] Taylor, M.D., International competition law: A new dimention for the WTO?. Cambridge University Press (2006).

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