An International Journal of Optimization
and Control: Theories & Applications
Vol.3, No.1, pp.1-5 (2013) © IJOCTA
ISSN:2146-0957 eISSN:2146-5703
DOI:10.11121/ijocta.01.2013.00122
http://www.ijocta.com

Mixed type second-order symmetric duality under F-convexity

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(Received March 31, 2012; in final form August 25, 2012)

Abstract. We introduce a pair of second order mixed symmetric dual problems. Weak, strong and converse duality theorems for this pair are established under $F-$convexity assumptions.

Keywords: Mixed integer programming; Second-order symmetric duality; $F-$convexity.
AMS Classification: 90C11, 90C46, 49N15.

1. Introduction

Dorn [5] introduced symmetric dual for quadratic programming problems. Subsequently, symmetric duality for nonlinear programming has been studied by many researchers [4, 9, 11]. Mangasarian [8] considered a nonlinear program and discussed second order duality under certain inequalities. Mond [10] established Mangasarian's duality relations assuming the kernel function to be bonvex/boncave.

The concept of mixed duality is interesting and useful both from theoretical as well as from algorithmic point of view. Bector et al. [3] introduced mixed symmetric dual models for a class of nonlinear multiobjective programming problems. Ahmad [1] studied invexity/generalized invexity for mixed type symmetric dual in multiobjective programming problems ignoring nonnegativity constraints of Bector et al. [3]. Recently, Ahmad and Husain [2] and Kailey et al. [6] discussed a pair of multiobjective mixed symmetric dual programs over arbitrary cones and established duality results under $K-$preinvexity/$K-$pseudoinvexity and $\eta-$bonvexity/$\eta-$pseudobonvexity assumptions respectively.

In this paper, we introduce a pair of second order mixed symmetric dual problems. Weak, strong and converse duality theorems for this pair are established under $F-$convexity assumptions.

2. Preliminaries

Let $\phi(x, y)$ be a real valued twice differentiable function defined on $R^n \times R^m$. Let $\nabla_x\phi(\bar{x}, \bar{y})$ and $\nabla_y\phi(\bar{x}, \bar{y})$ denote the gradient vector of ϕ with respect to x and y at $(\bar{x}, \bar{y})$. Also let $\nabla_{xx}\phi(\bar{x}, \bar{y})$ denote the Hessian matrix of $\phi(x, y)$ with respect to the first variable x at $(\bar{x}, \bar{y})$. The symbols $\nabla_{yy}\phi(\bar{x}, \bar{y})$, $\nabla_{xy}\phi(\bar{x}, \bar{y})$ and $\nabla_{yx}\phi(\bar{x}, \bar{y})$ are defined similarly.

Definition 1. *Let $S \subseteq R^n$. A functional $F : S \times S \times R^n \to R$ is sublinear if for any $(x,\ \bar{x}) \in S \times S$,*

(i) $F(x, \bar{x}; (a + b)) \leqq F(x, \bar{x}; a) + F(x, \bar{x}; b)$ for all $a, b \in R^n$,
(ii) $F(x, \bar{x}; \alpha a) = \alpha F(x, \bar{x}; a)$ for all $a \in R^n$ and

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for all $\alpha \in R_+$.

Definition 2. *The function* $\phi(., y)$ *is said to be second order* $F-$*convex at* $\bar{x}$, *for fixed* y, *if*

$$\phi(x,y) - \phi(\bar{x},y) + \tfrac{1}{2}p^T\nabla_{xx}\phi(\bar{x},x)p$$

$$\geqq F(x,\bar{x};\nabla_x\phi(\bar{x},y) + \nabla_{xx}\phi(\bar{x},y)p),\ \forall x \in R^n.$$

ψ *is second-order* $F-$ *concave at* $\bar{x} \in R^n$ *for fixed* y *if* $-\psi$ *is second-order* $F-$*convex at at* $\bar{x} \in R^n$ *for fixed* y.

3. Mixed Second Order Symmetric Dual Programs

For N= {1, 2, 3, ..., n} and M= {1, 2, 3, ..., m}, let $J_1 \subseteq N$, $K_1 \subseteq M$, $J_2 = N \setminus J_1$ and $K_2 = M \setminus K_1$. Let $|J_1|$ denote the number of elements in the set J_1. It may be noted that if $J_1 = \varnothing$, then $J_2 = N$, that is $|J_1| = 0$, $|J_2| = n$. Let $x^1 \in R^{|j_1|}$, $x^2 \in R^{|j_2|}$. Then it is clear that any $x \in R^n$ can be written as $x = (x^1, x^2)$. Similarly for $y^1 \in R^{|J_1|}$, $y^2 \in R^{|J_2|}$, any $y \in R^m$ can be written as $y = (y^1, y^2)$.

Now we formulate the following pair of mixed symmetric dual models and discuss the duality results.

Primal Problem (PP)

Minimize $H(x^1,x^2,y^1,y^2,p,r) = f(x^1,y^1) + g(x^2,y^2) - (y^1)^T[\nabla_{y^1}f(x^1,y^1) + \nabla_{y^1y^1}f(x^1,y^1)p]$
$-(y^2)^T[\nabla_{y^2}g(x^2,y^2) + \nabla_{y^2y^2}g(x^2,y^2)r]$
$-\frac{1}{2}p^T\nabla_{y^1y^1}f(x^1,y^1)p - \frac{1}{2}r^T\nabla_{y^2y^2}f(x^2,y^2)r$

subject to

$$\nabla_{y^1}f(x^1,y^1) + \nabla_{y^1y^1}f(x^1,y^1)p \leqq 0, \quad (1)$$
$$\nabla_{y^2}g(x^2,y^2) + \nabla_{y^2y^2}g(x^2,y^2)r \leqq 0, \quad (2)$$
$$(x^1,x^2) \geqq 0, \quad (3)$$

Dual Problem (DP)

Maximize $G(u^1,u^2,v^1,v^2,\ q,\ s) = f(u^1,v^1) + g(u^2,v^2) - (x^1)^T[\nabla_{x^1}f(u^1,v^1) + \nabla_{x^1x^1}f(u^1,v^1)q]$
$-(x^2)^T[\nabla_{x^2}g_i(u^2,v^2) + \nabla_{x^2x^2}g(u^2,v^2)s]$
$-\frac{1}{2}q^T\nabla_{x^1x^1}f(u^1,v^1)q - \frac{1}{2}s^T\nabla_{x^2x^2}f(u^2,v^2)s$

subject to

$$\nabla_{x^1}f_i(u^1,v^1) + \nabla_{x^1x^1}f(u^1,v^1)q \geqq 0, \quad (4)$$
$$\nabla_{x^2}g_i(u^2,v^2) + \nabla_{x^2x^2}g(u^2,v^2)s \geqq 0, \quad (5)$$
$$(v^1,v^2) \geqq 0, \quad (6)$$

where
(i) $f : R^{|J_1|}\times R^{|K_1|} \to R$ and $g : R^{|J_2|}\times R^{|K_2|} \to R$ are differentiable functions.
(ii) $p \in R^{|K_1|}$, $r \in R^{|K_2|}$, $q \in R^{|J_1|}$ and $s \in R^{|J_2|}$.

Theorem 1. (Weak Duality)
Let $(x^1,x^2,y^1,y^2,\ p,\ r)$ *be feasible for (PP) and* $(u^1,u^2,v^1,v^2,\ q,\ s)$ *be feasible for (DP). Let for sublinear functionals* $F_1 : R^{|J_1|} \times R^{|J_1|} \to R^{|J_1|}$, $F_2 : R^{|K_1|} \times R^{|K_1|} \to R^{|K_1|}$, $G_1 : R^{|J_2|} \times R^{|J_2|} \to R^{|J_2|}$ *and* $G_2 : R^{|K_2|} \times R^{|K_2|} \to R^{|K_2|}$,

(I) $F_1(x^1,u^1;a^1) + (a^1)^Tu^1 \geqq 0,\ \forall\ a^1 \in R_+^{|J_1|}$;
(II) $G_1(x^2,u^2;a^2) + (a^2)^Ty^2 \geqq 0,\ \forall\ a^2 \in R_+^{|K_1|}$;
(III) $F_2(v^1,y^1;b^1) + (b^1)^Tu^1 \geqq 0,\ \forall\ b^1 \in R_+^{|J_2|}$;
(IV) $G_2(v^2,y^2;b^2) + (b^2)^Ty^2 \geqq 0,\ \forall\ b^2 \in R_+^{|K_2|}$.

Suppose that

(i) $f(.,v^1)$ *is second-order* F_1-*convex at* u^1, *and* $f(x^1,.)$ *is second-order* F_2-*concave at* y^1,
(ii) $g(.,v^2)$ *is second-order* G_1-*convex at* u^2, *and* $g(x^2,.)$ *is second-order* G_2-*concave at* y^2.

Then,

$$H(x^1,x^2,y^1,y^2,\ p,\ r) \geqq G(u^1,u^2,v^1,v^2,\ q,\ s). \quad (7)$$

Proof. By second-order F_1-convexity of $f(.,v^1)$ at u^1, we have

$$f(x^1,y^1) - f(u^1,v^1) + \tfrac{1}{2}q^T\nabla_{x^1x^1}f(u^1,v^1)q$$

$$\geqq F_1(x^1,u^1;\nabla_{x^1}f(u^1,v^1) + \nabla_{x^1x^1}f(u^1,v^1)q).$$

Using hypothesis (I) and the dual constraint (4), we obtain

$$f(x^1,y^1) - f(u^1,v^1) + \tfrac{1}{2}q^T\nabla_{x^1x^1}f(u^1,v^1)q$$

$$\geqq -(u^1)^T(\nabla_{x^1}f(u^1,v^1) + \nabla_{x^1x^1}f(u^1,v^1)q). \quad (8)$$

By second-order F_2-concavity of $f(x^1,.)$ at y^1, we have

$$f(x^1,y^1) - f(x^1,v^1) - \tfrac{1}{2}p^T\nabla_{y^1y^1}f(x^1,y^1)p$$

$$\geqq F_1(x^1,u^1;-(\nabla_{y^1}f(x^1,y^1) + \nabla_{y^1y^1}f(x^1,y^1)p).$$

Using hypothesis (III) and the primal constraint (1), we obtain

$$f(x^1,y^1) - f(x^1,v^1) - \tfrac{1}{2}p^T\nabla_{y^1y^1}f(x^1,y^1)p$$

$$\geqq -(y^1)^T(\nabla_{y^1}f(x^1,y^1) + \nabla_{y^1y^1}f(x^1,y^1)p). \quad (9)$$

Combining inequalities (8) and (9), we have

$$f(x^1,y^1) - (y^1)^T(\nabla_{y^1}f(x^1,y^1) + \nabla_{y^1y^1}f(x^1,y^1)p)$$
$$-\frac{1}{2}p^T\nabla_{y^1y^1}f(x^1,y^1)p$$
$$\geqq f(u^1,v^1) - (u^1)^T(\nabla_{x^1}f(u^1,v^1) + \nabla_{x^1x^1}f(u^1,v^1)q)$$
$$-\frac{1}{2}q^T\nabla_{x^1x^1}f(u^1,v^1)q. \quad (10)$$

Similarly, by second-order G_1-convexity of $g(.,v^2)$ at u^2, second order G_2-concavity of $g(x^2,.)$ at y^2, hypothesis (II) and (IV), and constraints (2) and (5), we get

$$g(x^2,y^2) - (y^2)^T(\nabla_{y^2}f(x^2,y^2) + \nabla_{y^2y^2}f(x^2,y^2)q) -$$
$$\frac{1}{2}r^T\nabla_{y^2y^2}g(x^2,y^2)r$$
$$\geqq f(u^1,v^1)(u^2)^T(\nabla_{x^2}f(u^2,v^2) + \nabla_{x^2x^2}f(u^2,v^2)s)$$
$$-\frac{1}{2}s^T\nabla_{x^2x^2}f(u^2,v^2)s. \quad (11)$$

Adding inequalities (10) and (11), we obtain

$$f(x^1,y^1) + g(x^2,y^2) - (y^1)^T[\nabla_{y^1}f(x^1,y^1) + \nabla_{y^1y^1}f(x^1,y^1)p] - (y^2)^T[\nabla_{y^2}g(x^2,y^2) + \nabla_{y^2y^2}g(x^2,y^2)r] - \tfrac{1}{2}p^T\nabla_{y^1y^1}f(x^1,y^1)p$$
$$-\frac{1}{2}r^T\nabla_{y^2y^2}f(x^2,y^2)r$$

$$\geqq (u^1,v^1) + g(u^2,v^2) - (x^1)^T[\nabla_{x^1}f(u^1,v^1) + \nabla_{x^1x^1}f(u^1,v^1)q] - (x^2)^T[\nabla_{x^2}g(u^2,v^2) + \nabla_{x^2x^2}g(u^2,v^2)s] - \tfrac{1}{2}q^T\nabla_{x^1x^1}f(u^1,v^1)q$$
$$-\frac{1}{2}s^T\nabla_{x^2x^2}f(u^2,v^2)s$$

or

$$H(x^1,x^2,y^1,y^2,\ p,\ r) \geqq G(u^1,u^2,v^1,v^2,\ q,\ s).$$

Hence the result. □

Theorem 2. (Strong Duality)
Let $(\bar{x}^1,\bar{x}^2,\bar{y}^1,\bar{y}^2,\bar{p},\bar{r})$ be an optimal solution for (PP). Suppose that

(i) the matrices $\nabla_{y^1y^1}f(\bar{x}^1,\bar{y}^1)$, $\nabla_{y^2y^2}g(\bar{x}^2,\bar{y}^2)$ are non singular,

(ii) one of the matrices $(\partial/\partial y_i^1)(\nabla_{y^1y^1}f(\bar{x}^1,\bar{y}^1)$, $i = 1,\ 2,\ 3,\ ..,\ |K_1|$, and one of the matrices $(\partial/\partial y_i^2)(\nabla_{y^2y^2}g(\bar{x}^2,\bar{y}^2)$, $i = 1,\ 2,\ ..,\ |K_2|$ are positive or negative definite.

Then $(\bar{x}^1,\bar{x}^2,\bar{y}^1,\bar{y}^2,\bar{q} = 0,\bar{s} = 0)$ is feasible for (DP) and the corresponding objective function values are equal. If in addition the hypotheses of Theorem 1 hold for all feasible solutions of primal and dual problems, then $(\bar{x}^1,\bar{x}^2,\bar{y}^1,\bar{y}^2,\ \bar{q} = 0,\ \bar{s} = 0)$ is an optimal solution for (DP).

Proof. Since $(\bar{x}^1,\bar{x}^2,\bar{y}^1,\bar{y}^2,\ \bar{p},\ \bar{r})$ is an optimal solution of (PP), by the Fritz John necessary optimality conditions [7], there exist $\alpha \in R$, $\beta \in R^{|K_2|}$, $\gamma \in R^{|K_2|}$, $\eta_1 \in R^{|J_1|}$ and $\eta_2 \in R^{|J_2|}$ such that the following conditions are satisfied at $(\bar{x}^1,\bar{x}^2,\bar{y}^1,\bar{y}^2,\ \bar{q},\ \bar{s})$:

$$\alpha(\nabla_x^1 f(\bar{x}^1,\bar{y}^1)) + \nabla_{y^1x^1}f(\bar{x}^1,\bar{y}^1)[\beta - \alpha\bar{y}^1]$$
$$+\nabla_{x^1}(\nabla_{y^1y^1}f(\bar{x}^1,\bar{y}^1)\bar{p})[\beta - \alpha(\bar{y}^1 + \frac{1}{2}\bar{p})] - \eta_1 = 0, \quad (12)$$

$$\alpha(\nabla_x^2 g(\bar{x}^2,\bar{y}^2)) + \nabla_{y^2x^2}g(\bar{x}^2,\bar{y}^2))[\beta - \alpha\bar{y}^2]$$
$$+\nabla_{x^2}(\nabla_{y^2y^2}g(\bar{x}^2,\bar{y}^2)\bar{r})[\beta - \alpha(\bar{y}^2 + \frac{1}{2}\bar{r})] - \eta_2 = 0, \quad (13)$$

$$\nabla_{y^1y^1}f(\bar{x}^1,\bar{y}^1)[\beta - \alpha(\bar{y}^1 + \bar{p}]$$
$$+\nabla_{y^1}(\nabla_{y^1y^1}f(\bar{x}^1,\bar{y}^1)\bar{p})[\beta - \alpha(\bar{y}^1 + \frac{1}{2}\bar{p})] = 0, \quad (14)$$

$$\nabla_{y^2y^2}g(\bar{x}^2,\bar{y}^2)[\beta - \alpha(\bar{y}^2 + \bar{r}]$$
$$+\nabla_{y^2}(\nabla_{y^2y^2}g(\bar{x}^2,\bar{y}^2)\bar{r})[\gamma - \alpha(\bar{y}^2 + \frac{1}{2}\bar{r})] = 0, \quad (15)$$

$$\nabla_{y^1y^1}f(\bar{x}^1,\bar{y}^1)[\beta - \alpha(\bar{y}^1 + \bar{p}] = 0, \quad (16)$$
$$\nabla_{y^2y^2}g(\bar{x}^2,\bar{y}^2)[\gamma - \alpha(\bar{y}^2 + \bar{r}] = 0, \quad (17)$$
$$\beta^T[\nabla_{y^1}f(\bar{x}^1,\bar{y}^1) + \nabla_{y^1y^1}f(\bar{x}^1,\bar{y}^1)\bar{p}] = 0, \quad (18)$$
$$\gamma^T[\nabla_{y^2}g(\bar{x}^2,\bar{y}^2) + \nabla_{y^2y^2}g(\bar{x}^2,\bar{y}^2)\bar{r}] = 0, \quad (19)$$
$$\eta_1^T\bar{x}^1 = 0, \quad (20)$$
$$\eta_2^T\bar{x}^2 = 0, \quad (21)$$
$$(\alpha,\beta,\gamma,\eta_1,\eta_2) \geqq 0, \quad (22)$$
$$(\alpha,\beta,\gamma,\eta_1,\eta_2) \neq 0. \quad (23)$$

Using hypothesis (i), equations (16) and (17) imply

$$\beta = \alpha(\bar{y}^1 + \bar{p}), \quad (24)$$
$$\gamma = \alpha(\bar{y}^2 + \bar{r}). \quad (25)$$

Now suppose, $\alpha = 0$. Then equation (24) and(25) imply $\beta = 0$, $\gamma = 0$, which along with equations (12) and (13) yield $\eta_1 = 0$, $\eta_2 = 0$.

Thus $(\alpha, \beta, \gamma, \eta_1, \eta_2) = 0$, which contradicts (25). Hence

$$\alpha > 0. \tag{26}$$

Now using equations (24) and (25) in (14) and (15), we get

$$\nabla_{y^1}(\nabla_{y^1y^1} f(\bar{x}^1, \bar{y}^1)\bar{p})[\alpha\bar{p}] = 0,$$
$$\nabla_{y^2}(\nabla_{y^2y^2} g(\bar{x}^2, \bar{y}^2)\bar{r})[\alpha\bar{r}] = 0.$$

Therefore hypothesis (ii) and (26) yield

$$\bar{p} = 0, \tag{27}$$
$$\bar{r} = 0. \tag{28}$$

From (24), (25), (27) and (28), we get

$$\beta = \alpha\bar{y}^1, \tag{29}$$
$$\gamma = \alpha\bar{y}^2. \tag{30}$$

Using (26), (27) and (29) in (12), we obtain

$$\alpha[\nabla_{x^1} f(\bar{x}^1, \bar{y}^1)] - \eta_1 = 0,$$

or

$$\nabla_{x^1} f(\bar{x}^1, \bar{y}^1) = \frac{\eta_1}{\alpha} \geqq 0, \tag{31}$$

and

$$(x^1)^T[\nabla_{x^1} f(\bar{x}^1, \bar{y}^1)] = \frac{(x^1)^T\eta_1}{\alpha} = 0, \tag{32}$$

(using equation (20)).

Further, from (26), (28) and (30), we get

$$\alpha[\nabla_{x^2} g(\bar{x}^2, \bar{y}^2)] - \eta_2 = 0,$$

or

$$\nabla_{x^2} g(\bar{x}^2, \bar{y}^2) = \frac{\eta_2}{\alpha} \geqq 0, \tag{33}$$

and

$$(x^2)^T[\nabla_{x^2} f(\bar{x}^1, \bar{y}^1)] = \frac{(x^2)^T\eta_2}{\alpha} = 0, \tag{34}$$

(using equation (21)).

Finally, from (29) and (30),

$$\bar{y}^1 \geqq 0 \text{ and } \bar{y}^2 \geqq 0.$$

Thus $(\bar{x}^1,\ \bar{x}^2,\ \bar{y}^1,\ \bar{y}^2,\ \bar{q} = 0,\ \bar{s} = 0)$ satisfies the dual constraints (4)-(6), and so it is a feasible solution for the dual problem (DP).

Now using (26), (27), (29) in (18), we obtain

$$(y^1)^T[\nabla_{y^1} f(\bar{x}^1, \bar{y}^1)] = 0, \tag{35}$$

Similarly, using (26), (28), (30) in (19), we get

$$(y^2)^T[\nabla_{y^2} f(\bar{x}^1, \bar{y}^1)] = 0, \tag{36}$$

Therefore, using (27), (28), (32) and (34)- (36), we get

$$\begin{aligned}
&H(\bar{x}^1,\ \bar{x}^2,\ \bar{y}^1,\ \bar{y}^2,\ \bar{p} = 0,\ \bar{r} = 0)\\
&=f(\bar{x}^1, \bar{y}^1) + g(\bar{x}^2, \bar{y}^2) - (\bar{y}^1)^T[\nabla_{\bar{y}^1} f(\bar{x}^1, \bar{y}^1)\\
&\quad + \nabla_{y^1y^1} f(\bar{x}^1, \bar{y}^1)\bar{p}] - (\bar{y}^2)^T[\nabla_{y^2} g(\bar{x}^2, \bar{y}^2)\\
&\quad + \nabla_{y^2y^2} g(x^2, y^2)\bar{r}] - \frac{1}{2}\bar{p}^T\nabla_{y^1y^1} f(\bar{x}^1, \bar{y}^1)\bar{p}\\
&\quad - \frac{1}{2}\bar{r}^T\nabla_{y^2y^2} f(\bar{x}^2, \bar{y}^2)\bar{r}\\
&=f(\bar{x}^1, \bar{y}^1) + g(\bar{x}^2, \bar{y}^2) - (\bar{x}^1)^T[\nabla_{x^1} f(\bar{x}^1, \bar{y}^1)\\
&\quad + \nabla_{x^1x^1} f(\bar{x}^1, \bar{y}^1)\bar{q}] - (\bar{x}^2)^T[\nabla_{x^2} g(\bar{x}^2, \bar{y}^2)\\
&\quad + \nabla_{x^2x^2} g(\bar{x}^2, \bar{y}^2)\bar{r}] - \frac{1}{2}\bar{q}^T\nabla_{x^1x^1} f(\bar{x}^1, \bar{y}^1)\bar{q}\\
&\quad - \frac{1}{2}\bar{s}^T\nabla_{x^2x^2} f(\bar{x}^2, \bar{y}^2)\bar{s}\\
&=G(\bar{x}^1,\ \bar{x}^2,\ \bar{y}^1,\ \bar{y}^2,\ \bar{q} = 0,\ \bar{s} = 0).
\end{aligned}$$

That is, the two objective function values are equal. By using weak duality it can be easily shown that $(\bar{x}^1, \bar{x}^2, \bar{y}^1, \bar{y}^2,\ \bar{q} = 0,\ \bar{s} = 0)$ is an optimal solution for (DP).

□

Theorem 3. (Converse Duality)
Let $(\bar{u}^1, \bar{u}^2, \bar{v}^1, \bar{v}^2,\ \bar{q}, \bar{s})$ be an optimal solution for (DP). Suppose that

(i) the matrices $\nabla_{x^1x^1} f(\bar{u}^1, \bar{v}^1)$, $\nabla_{x^2x^2} g(\bar{u}^2, \bar{v}^2)$ are non singular,
(ii) one of the matrices $(\partial/\partial x_i^1)(\nabla_{x^1x^1} f(\bar{u}^1, \bar{v}^1)$, $i = 1,\ 2,\ 3, .., |J_1|$, and one of the matrices $(\partial/\partial y_i^2)(\nabla_{y^2y^2} g(\bar{u}^2, \bar{v}^2)$, $i = 1,\ 2,\ ..,\ |J_2|$ are positive or negative definite.

Then $(\bar{u}^1, \bar{u}^2, \bar{v}^1, \bar{v}^2, \bar{p} = 0, \bar{r} = 0)$ is feasible for (PP) and the corresponding objective function values are equal. If in addition the hypotheses of Theorem 1 hold for all feasible solutions of primal and dual problems, then $(\bar{u}^1, \bar{u}^2, \bar{v}^1, \bar{v}^2,\ \bar{p} = 0,\ \bar{r} = 0)$ is an optimal solution for (PP).

Proof. The proof follows on the lines of Theorem 2.

□

Acknowledgments

The second author is thankful to the MHRD, Government of India for providing financial support.

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