

MORPHOLOGICAL IMAGE PROCESSİNG WITH FUZZY LOGIC

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Bulanık Mantık ile Morfolojik Görüntü İşleme

ÖZET

Bu çalışmada bulanık mantık ile morfolojik görüntü işleme teknikleri incelenmiştir. Matematiksel morfoloji işlemleri incelenmiş buna uygun olarak da bulanık mantık ve bulanık kümeler teoremi kullanılarak deneysel bulanık morfoloji işlemleri tanımlanmıştır. Matematiksel morfoloji işlemleri genel olarak kümeler ve küme işlemlerine benzer şekilde tanımlandığından bulanık kümeler teorisini matematiksel morfolojiye uygulamak o derece kolaylaşmıştır. Bulanık küme teorisine uygun olarak tanımlanan bulanık morfoloji işlemleri iki aşamada gerçekleştirilmiştir. Birinci aşama bulanıklaştırma üyelik fonksiyonunun tanımlanması ikinci aşama ise alpha kesmeleri ile birinci aşamada tanımlanan bulanıklaştırma fonksiyonu üzerinden işlemlerin gerçekleştirilmesidir. Bu kuramla görüntü işleme tekniklerine yeni boyutlar katılmıştır. Özellikle siyah-beyaz görüntülerdeki yapılar ayrık bir yapı olduğundan yani 1 ve 0 lardan oluşan birer küme olduklarından bulanıklaştırma işlemleri için güzel bir uygulamadır. Bu çalışmada siyah-beyaz bir görüntü kullanılarak SAKAWA ve YUMINE'nin bulanık üyelik fonksiyonları ile bulanıklaştırılma işlemleri gerçekleştirilmiş ve en temel matematiksel morfoloji işlemleri olan "EROSION" ve "DILATION" alpha kesimleri üzerinden deneysel olarak irdelenmiştir.

Anahtar Kelimeler: Bulanık Kümeler Teorisi, Bulanık Mantık, Matematiksel Morfoloji, Bulanık Morfoloji

ABSTRACT

In this paper, fuzzy morphological image processing is explained and reviewed. Based on the mathematical morphology rules, fuzzy sets and fuzzy logic theorem fuzzy morphology operations are defined. In general mathematical morphology operation definitions are similar structures set theory and set operations definitions. For this reason fuzzy set theory is easily applied to the mathematical morphology. Fuzzy morphology operations are defined and implemented in two steps. Initial step is the fuzzification process which are constructed over the fuzzy membership functions. Second step is the realization process of the fuzzification process via the alpha cuts of the fuzzy membership functions. With the help of this theorem image processing techniques gain many opportunities for different operations. Since the gray scale images are discrete structures which have 1 and 0 sets, fuzzification process is a good application for transforming the discrete set to the fuzzy set. In this study a gray scale image is fuzzified according to the SAKAWA's and YUMINE's fuzzy membership functions. Basic mathematical morphology operations which are "EROSION" and "DILATION" implemented and inspected via the fuzzy membership functions alpha cuts.

Keywords: Fuzzy Sets Theory, Fuzzy Logic, Mathematical Morphology, Fuzzy Morphology

1. INTRODUCTION

Mathematical morphology is a collection of operations which produces useful outcomes in image processing area. It is completely based on set theory. For this reason all of the operations in morphology are defined on the simple set operation rules to apply them on image pixels. By knowing this fact there has been many approaches proposed to the morphology to extend its applications. Because the mathematical morphology is originally defined on binary images, most of the theories for extending the mathematical

morphology tried to broaden this fact to the grayscale and colorscale images. Some of these theories were the *umbra approach*, *threshold set approach* and *complete lattice approach*. Some of these approaches extended the mathematical morphology but because of the fundamental set theory is split in these theories, they had some drawbacks applicationwise. But one of the approaches which was not counted above is *Fuzzy Logic Approach*.

In this approach fundamentals of fuzzy set theory and the fuzzy logic is used to define the mathematical

Erosion is denoted as $f \ominus b$ is defined as:

$$(f \ominus b)(s,t) = \min\{f(s+x,t+y) - b(x,y) \mid (s+x),(t+y) \in D_f; (x,y) \in D_b\}$$

where D_f and D_b are the domains of f and b , respectively.[2] The conditions that $(s+x)$ and $(t+y)$ have to be in the domain of f , and x and y have to be in the domain of b , is analogous to the condition in the binary definition of erosion, where structuring element has to be completely contained by the set being eroded. For further details please check the *Gonzales Image Processing Book*. [2].

Mathematical morphology can be used as the basis for developing image segmentation procedures with a wide range of applications and it also plays a major role in procedures for image description.

2.FUZZY SETS AND FUZZY LOGIC

Fuzzy Logic is a means of dealing with information in the same way that humans or animals do. Fuzzy Logic is built around the concept of reasoning in degrees, rather than in boolean (yes/no 0/1) expressions like computers do.

Variables are defined in terms of fuzzy sets. Rules are specified by logically combining fuzzy sets. The combination of fuzzy sets defined for input and output variables, together with a set of fuzzy rules that relate one or more input fuzzy sets to an output fuzzy set, which built a fuzzy system. [5]

Fuzzy systems represent well-defined static deterministic functions. Therefore reaction of a fuzzy system to inputs is anything but fuzzy. Inputs are presented to the system as specific values, and the fuzzy system produces a specific output value. The operation of a fuzzy system is thus analogous to that of conventional systems.

Most of the fuzzy systems are for to use in decision making or pattern recognition applications. An ordinary set splits the data into those items that are completely in the set and those items that are completely outside of the set.[5]

To understand it easily it can be explained by assigning the value 1 to all those data which are members of the set and the value 0 to all data which are not members of the set. For ordinary sets, only these two values is called the characteristic function of the set. In the following Figure-2 we can see one of the application areas of fuzzy sets and fuzzy logic as applied on image processing area in steps. [6]. *Fuzzy image processing is the collection of all approaches that understand, represent and process the images, their segments and features as fuzzy sets. The representation and processing depend on the selected fuzzy technique and the problem to be solved.*[6]

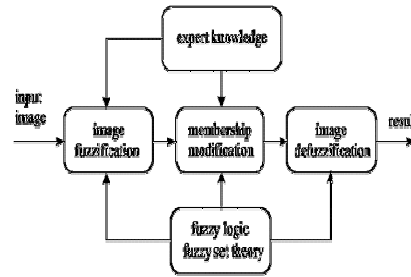


Figure 2. Fuzzy Logic Theorem in Image Processing

Fuzzy sets allow the possibility of the degrees of membership. That is any of the values between 0 and 1 (including 0 and 1) may be assigned. For example lets assume a given fuzzy set be “fast cars” set. It is possible a car can be a member of this set or not. This would be a rather fast car, but not the fastest car imaginable. The function which assigns this value is called the membership function associated with the fuzzy set. Fuzzy membership functions are the mechanism through which the fuzzy system interacts with the outside world. The range, or possible output values of a membership function is the interval [0,1], the set of all real numbers between 0 and 1, inclusive. A typical choice for a fuzzy membership function is a piecewise linear trapezoidal function. [5].

3.FUZZY MORPHOLOGY

Conventional mathematical morphology provide many ways to introduce fuzzy concepts within its definitions. The most applicable area is the signals, which can be morphologically processed. The structuring elements are used for probing the signals as a structural parameter, and many of the set concepts within the primitive definitions of the mathematical morphology. Some of these definitions can be counted as Minkowski addition, partial order relations and ranking, suprema and infima and so forth.

Concerning images, three points of fuzziness emerged to the image processing area. A grayvalue image is a fuzzy set in the sense that it is a fuzzy version of a binary image, or that a grayvalue represents the degree to which pixel belongs to the image foreground. [4].

The first point of the fuzzification of the images is that a grayvalue pixel at a position $g(x,y)$ defines the degree to which a pixel actually exists in the image. This will always create a renormalization of the grayvalues because they usually cover a range of integer values starting from 0 to 255, but membership degrees are to be expected from the range [0,1]. It is only understandable from the membership degrees is that the pixel at position (x,y) belongs to the degree $g(x,y)/g_{max}$ to the image. Defuzzification of the image is binary. But we can not determine which type of degree that pixel position belongs to because it is not deterministic. The following Figure-3 gives us

opportunity to understand how does the fuzzification and defuzzification of the image is done. [6]

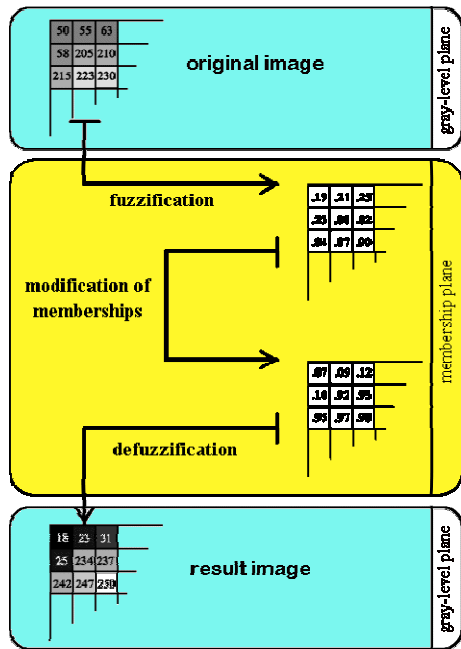


Figure 3. Fuzzification and Defuzzification Process on Image Processing

This general idea creates some problem of selection of the foreground and background problem because in fuzzy systems it is always possible to assign a noisy pixel point assigned either to the foreground or background object recognition tasks.

In the second approach positional labeling of the objects as a pixel group is another important factor. Fuzzification in this approach helps us on determination of the specific fuzzy borders of the foreground or specifying an occluded object from the content of the image.

As a third point which is not stated above is the fuzzy pixel. Fuzziness mean that the relationship between the spatial position of the pixel can be unclear for various reason (image acquisition process involves not correct geometric positioning) and its assigned pixel value. [4].

From all of the stated points above why is fuzzification important for images while there has been some drawbacks. They are many reasons that can be stated as answers but the most important two are [6]:

- Fuzzy techniques are powerful tools for knowledge representation and processing
- Fuzzy techniques can manage the vagueness and ambiguity efficiently

There are three important problems in which the image processing is fuzzy naturally itself. For example grayness ambiguity, geometrical fuzziness, Vague(complex/ill-defiend) knowledge are some of those. The below Figure-4 gives some idea about the fuzzy problem nature in image processing [6].

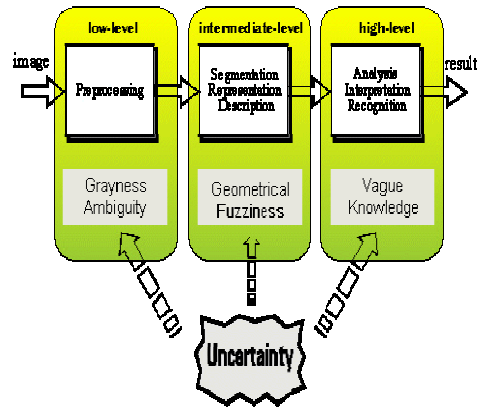


Figure 4. Uncertainty/imperfect knowledge in image processing

The application of fuzziness in mathematical morphology applied to the image pixels and the structural element in order to perform the morphological operations. In the following sections there are many different approaches of fuzzy mathematical morphological approaches will be specified.

The order of the fuzzy morphology approaches that will be explained in detail which are applied on the erosion and dilation operations are as follows: Alpha-Morphology mixed with Bloch and Maitre-Morphologies, Sinha and Dougherty-Morphologies. Other morphological structures which are Fuzzy S and T-Norms, Fuzzy Ranking and Fuzzy Fusion approaches can be learned from the *A Tutorial on Fuzzy Morphology* paper for further details [4]. They were not explained in this paper since fuzzy morphology is a design issue that there exists many ways that can be applied to the mathematical approaches. For instance in this paper there is a trial method of erosion and dilation operation is designed with the help of *Sakawa and Yumine's* fuzzy membership function.

3.1.Alpha-Morphology

According to the Bloch and Maitre the initial trial who combined the mathematical morphology and fuzzy logic was Kaufmann in 1988. Kaufmann proposed alpha-cuts approach for further fuzzy set operations. Fuzzy set that contains all elements with a membership of $\alpha \in [0,1]$ and above is called the *-cut*

of the membership. The membership function is cut horizontally at a finite number of α -levels between 0 and 1. For each α -level of the parameter, the model is run to determine the minimum and maximum possible values of the output. This information is then directly used to construct the corresponding fuzziness (membership function) of the output which is used as a measure of uncertainty. Based on this fact a fuzzy set maximum and minimum operation can be defined over α -cuts.

α -cuts provide a appropriate way of combining a fuzzy set and a crisp set. Given a fuzzy set $\mu(x)$ where x is an element of the universe of discourse X and assigning membership degrees from the intervall $[0,1]$ to each element of X , then for $0 \leq \alpha \leq 1$, the α -cut of $\mu(x)$ is the set of all $x \in X$ with membership degree at least as large as α [4]:

$$\mu_\alpha = \{x \mid \mu(x) \geq \alpha\}$$

By the help of this definition of α -cuts fuzzy set operations are easily derived from the crisp counterparts. For example a union set operation of two fuzzy sets $\mu(x)$ and $\nu(x)$ is defined as

$$[\mu \cup \nu](x) = \max[\mu(x), \nu(x)] \text{ which implies } (\mu \cup \nu)_\alpha = \mu_\alpha \cup \nu_\alpha$$

For defining the Minkowski addition of two sets A and B translate $\tau_\alpha(X)$ of a (crisp) set X by a vector α has to be considered:

$\tau_\alpha(X) = \{y \mid y = x - \alpha, x \in X\}$ implies to the definition of two sets A and B is given by

$$A \oplus B = \{\alpha \in A \mid \tau_\alpha(B) \cap A \neq \emptyset\}$$

Moreover if A and B are given graphically, the shape of A is “dilated” by the shape of set B . As a dual operation, the Minkowski subtraction of a set B from a set A can be defined as well. Now only elements of A belong to the result set, if their corresponding translate B completely belongs to A :

$$A \ominus B = \{\alpha \in A \mid \tau_\alpha(B) \subseteq A\}$$

This operation reminds the “eroding” the shape of A by the shape of B . By using these ideas we can define the erosion and dilation operations over α -cuts. To realize this we, have to fuzzify the image pixels to create a fuzzy set image. With the help of this fuzzy set image we will define a α -cut from that fuzzy set image as a threshold value. So the image $f(x,y)$ will be defined as $g_\alpha(x)$. This makes us to remember the scale of a grayscale image has been selected over the threshold α . Then if we define a structuring element mask for instance a 3x3 fuzzified weighted mask

$\mu_\alpha(x)$ over the α -cut. The definition of a 2-D α -cut Bloch and Maitre dilation and erosion as follows with the help of the Minkowski addition and subtraction [4]:

$$[g(x) \oplus \mu(x)]_\alpha(x) = \sup_{y \in X} \min[g(x-y), \mu(y)] \text{ where } y \in X: \text{Dilation}$$

$$[g(x) \ominus \mu(x)]_\alpha(x) = \inf_{y \in X} \max[g(x-y), \mu(y)] \text{ where } y \in X: \text{Erosion}$$

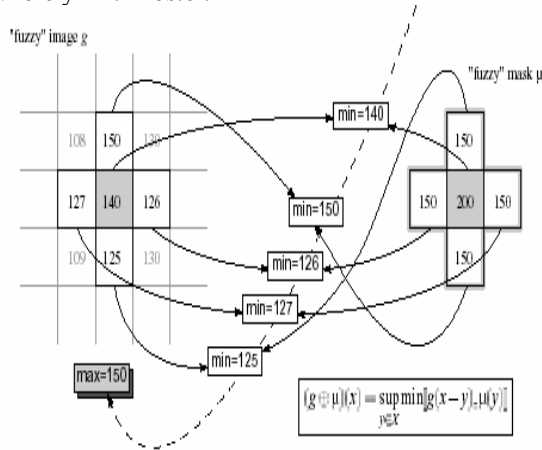


Figure 5. Understanding the Bloch-Maitre Formula on Erosion-Dilation Operations [4]

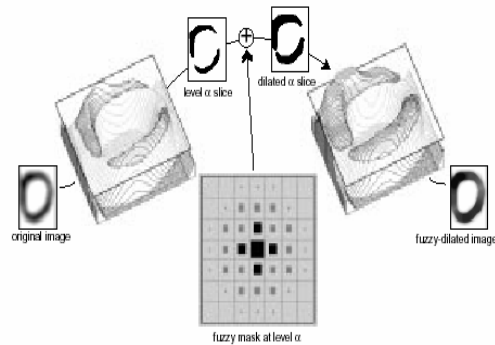


Figure 6. A fuzzy dilation with an α -cut fuzzy structuring element mask [4]

3.2. The Sinha and Dougherty Morphologies

Sinha and Dougherty proposed the first group of morphologies which broadend the concept of α -cut morphologies. The base rule of these morphological approach can be found in the translation of τ_x of a set B and the degree, to which it is covered by the set A . For the Minkowski subtraction it is required that $\tau_x(B) \subseteq A$ for taking x into the result set. But in this approach there exist a drawback of x belonging to the result set fails even all the elements besides of only one of the translated B belong to A . To resolve this drawback it can be used a more flexible measure for “subsethood”, which is 1 for the case of crisp subsethood.

Sinha and Dougherty proposed “inclusion indicator”. Inclusion indicator represents the degree of a fuzzy set which is a subset of another fuzzy set. Some of the necessary rules are stated below and they were updated by Popov[4]. The fuzzy subsets of Universal set U shall be denoted by $F[U]$ and $FR[U \times V]$ is the class of the fuzzy relations in $U \times V$ [1]. When it is said that $R \in FR(F(U) \times F(U))$ is an inclusion grade between fuzzy subsets when the following axioms in $U \times V$ [Fuzzy Morphological Operators]:

1. $R(A, B) = 1 \Leftrightarrow A \subseteq B$
2. $R(A, B) = 0 \Leftrightarrow \exists x \in U$ such that $A(x) = 1$ and $B(x) = 0$
3. R is non decreasing in its second argument ($B \subseteq C \Rightarrow R(A, B) \leq R(A, C)$)
4. R is non increasing in its first argument ($B \subseteq C \Rightarrow R(C, A) \leq R(B, A)$)
5. $R(A, B) = R(BC, AC)$
6. $R(A \cup B, C) \geq \min(R(A, C), R(B, C))$
7. $R(A, B \cap C) \geq \min(R(A, B), R(A, C))$

All of these axioms are conformant with the ordinary set operations. A fuzzy erosion and dilation can be constructed by providing the result value at image position x as.

$[g \oplus \mu](x) = [g', (-\mu)']$ where g' represents complement of g and $I(A, B) = 1$ iff $B \subseteq A$ and 0 otherwise: *Dilation*

$[g \ominus \mu](x) = I[\tau x(\mu), g]$ where $I(A, B) = 1$ iff $B \subseteq A$ and 0 otherwise: *Erosion*



Figure 7. Initial Image

Figure 8. Erosion

Figure 9. Dilation

The Figures 7-8-9 shows the applications Erosion and Dilation of the Sinha and Dougherty morphologies on the initial image [1].

4. IMPLEMENTED METHODS

Trial Fuzzy Erosion and Dilation Work with Sakawa and Yumine’s Fuzzy Membership Function

Anyone can design a fuzzy morphology by knowing some of the construction principles as follows [4]:

- If there is a dilation \oplus , construct its dual erosion by $g \ominus \mu = (g' \oplus \mu)'$ and vice versa.

- Replace occurrences of maximum and minimum operators by s- and t-norms.
- Replace supremum or infimum selections by argmax or argmin selections.
- Replace subset relation by inclusion indicators.
- Replace union by s-norms, sections by t-norms and complements by fuzzy complements.
- Use several s-, t-norms or fuzzy complements within the same expression.
- Replace ranking by dominance degrees.
- Fuse fuzzy values by fuzzy norms or fuzzy integrals to yield final results.
- Decompose a fuzzy set into its α -cuts, perform an operation there and recombine the result from the processed α -cuts.
- Adapt the construction to practical needs.
- Employ alternative definitions of the same operation in the non-fuzzy case to obtain different fuzzy operations.

These rules helps us to design a fuzzy erosion and dilation operation by using different approaches of fuzzy sets and fuzzy logic rules. For instance one of the Fuzzy Set Membership function which is defined by Sakawa and Yumine’s membership function can be used as a structuring mask to convolve with a fuzzified version of an input image. By performing this approach someone can define a fuzzy erosion and dilation image. Sakawa and Yumine’s proposed fuzzy membership functions are as follows:

- $\mu(x) = c * (1 - \exp(-(b-x)/(b-a)))$ where $x \in [a, b]$ and $c \in R$
- $\mu(x) = 0.5 + c * \arctan(1/\exp(-a*(x-b)))$

By using these membership functions we can create a structuring element of 3x3 size mask filter for performing an α -cut morphology erosion and dilation can be defined as follows:

$[g(x) \oplus \mu(x)] \alpha(x) = \sup \min[g(x-y), \mu(x)]$ where $g(x)$ is input image and $y \in X$: *Dilation*

$[g(x) \ominus \mu(x)] \alpha(x) = \inf \max[g_{\max} - g(x-y), \mu(x)]$ where $g(x)$ is input image and $y \in X$: *Erosion*

In this approach if we think of the image matrix as $g(x, y)$ it can be initially fuzzified by $g(x, y)/g_{\max}$ then with the help of the structuring element Sakawa and Yumine’s $\mu(x)$ it can be used for the above defined dilation and erosion definitions. Since above definitions have been selected as α -cut then the α can be selected as $\alpha \in [g(0, 0)/g_{\max}, g[n, n]/g_{\max}]$ so that the result can be observed for each case of individual threshold α .

5. RESULTS

With the help of Sakawa and Yumine’s Fuzzy membership functions, fuzzy morphology operations are performed. A gray scale image is fuzzified with fuzzy membership functions. Then a fuzzy structuring element is traversed on the whole image to process erosion and dilation operations. Since structuring element is a mask, operations are processed via the convolution basis. Structuring element must also be fuzzified during the convolution process. Figure in below shows the structuring element used in the operations.

0	150	0
150	200	150
0	150	0

Figure 10. Structuring Element Mask

Structuring element is selected as a 3x3 mask matrix to cover the whole image boundaries. Generally in image processing odd sized mask shows a pixel values neighboring pixels in general. As the size of the mask increases, the detailed results of the operations decreases. For this reason it is better to have a small odd sized structuring element mask for better performance. The values used inside the structuring element are the pixel values. These pixel values are randomly selected values so a user can define different elements for the mask. As a result of the convolution process following images (seen in Figure-11-12-13) are generated based on the fuzzy morphological operations: erosion and dilation



Figure 11. Input Image



Figure 12. Dilation



Figure 13. Erosion

6.CONCLUSION

From all the information stated above fuzzy logic and fuzzy set theory provide many solutions to the mathematical morphology algorithms. They have extended way of processing the grayscale images by the help of the fuzzy morphological operators. Fuzzy set and fuzzy logic theory is a new research area for defining new algorithms and solutions in mathematical morphology environment.

Besides the fuzzy morphology there are other various proposals of generalized morphologies in another context. For instance 3-D morphologies, geodesic morphologies, multivariate (color) morphologies, conditional morphologies or statistical morphologies are some of them.

The objective of this research is to show a path to the researchers that there exist other methods in mathematical morphology literature. So that they can devise new algorithms and implementations in image processing applications.

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