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REPRESENTATION METHOD EFFECTS ON VIBRATIONAL GENETIC ALGORITHM IN 2-D AIRFOIL DESIGN

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ABSTRACT
In this article, two different curve representation methods; Parsec and Bezier representation methods are tested via vibrational genetic algorithm [VGA] to show the effect of representation method on search type optimization process in 2-D airfoil design. From the results obtained, it is concluded that Parsec method has a better performance in subsonic flow conditions within the inverse design problem. On the other hand, it is also concluded that Bezier representation method is more efficient than Parsec in transonic flow regime.

Keywords: *Genetic algorithm, Parsec, Bezier, Optimization*

İKİ BOYUTLU KANAT PROFİLİ TASARIMINDA GEOMETRİ TEMSİL YÖNTEMİNİN TİTREŞİMLİ GENETİK ALGORİTMA ÜZERİNE ETKİLERİ

ÖZET
Bu makalede iki boyutlu kanat tasarımı dahilinde kanat profil geometrisi temsil yönteminin optimizasyon sürecine etkileri test edilmiştir. Temsil yöntemleri olarak Parsec formülasyonu ve Bezier parametrik eğri yaklaşımı, optimizasyon yöntemi olarak ise reel kodlu titreşimli genetic algoritma dikkate alınmıştır. Yapılan çalışma sonucu Parsec yönteminin ses altı akış şartlarında tersten tasarım probleminde daha kısa sürede optimizasyona imkan sağladığı,buna karşılık Bezier parametrik eğri yönteminin ise ses civarı akış şartlarında daha çabuk yakınsamaya olanak verdiği gözlemlenmiştir.

Anahtar Kelimeler: *Genetik Algoritma, Parsec, Bezier, Optimizasyon*

1. INTRODUCTION

The main goal in aerodynamics is commonly to increase the efficiency like the ratio of lift over drag. Aircraft wings are the primary subject to optimization efforts. Airfoils are the basic elements of wing geometry; they determine a large share of wing flow phenomena though they are just two-dimensional [2D] sections of the physical wing surface. Given a designer's refined knowledge about the occurring flow phenomena, his goal may be to obtain certain pressure distributions on wing surfaces: This may be reached by inverse approaches with a shape resulting from the effort, or by applying optimization strategies to drive results toward ideal values [1]. In an aerodynamic optimization problem the optimum design is an unknown shape, and the performance of the optimization process depends on how well the geometry representation method can approach the optimum shape.

To pose the airfoil shape optimization problem, the design variables that control the geometrical shape of airfoil are needed. The goal is to propose functions with a minimum set of input parameters for shape variation, function structure and their parameters chosen to address special aerodynamic or fluid mechanic phenomena. However, a method with more design parameters should have a more complete set of shapes, and therefore can approach the design target better [2].

There are different functions to describe airfoil sections. In addition to well known airfoil descriptions the aircraft industry has also developed their own mathematical tools to shape specific wing and blade sections. Among these geometrical representations it is possible to make a categorization depending on the type of the mathematical functions. The classification can be divided into three categories; polynomial function based representation methods, sinusoidal function based representation methods, and others. Bezier functions [3], NURBS [4], Parsec method based functions [5], and naca 4- and 5-digit series functions [6] are well known polynomial type representation methods of airfoil sections. B-spline functions [7] are also other well known type of polynomial functions. Mathematically, any continuous function defined on a closed interval can be represented by an infinite series of normal modes which form a complete set of bases. The set of Fourier sine functions is an example of such a complete set. There are several well known shape functions for wing section modifications. Hicks-Henne functions, Wagner functions, and aerofunctions are some of them [8]. Joukowski transformation, mesh point and grid parameterization methods may be given examples of other methods category [9]. In mesh pint method for a numerical computation, mesh points are used to represent the airfoil surface. The mesh-point method uses the coordinates of these points directly as the design parameters.

Different representation methods may have different performances depending on the optimization algorithms. There are some studies which include the comparisons among representation methods related to gradient based optimization algorithms [10, 11]. Both studies showed that the mesh-point method can reach the highest accuracy (lowest cost function value) among other methods such as Parsec, Hicks-Henne, B-spline, and the Class function / Shape function Transformation. This is an expected result; because the accuracy is proportional to the number of design parameters. More design parameters usually mean a more complete design space and hence a better capability of approaching the design target. More design parameters mean also more sensitivity to local perturbations. Gradient based optimization algorithms are mainly based on local sensitivities related to perturbations [12].

On the other hand, conventional gradient-based algorithms may be ineffective in some optimization problems with nondifferentiable, highly nonlinear, and many local minima cost functions because of local minimums or the difficulty in calculating gradients. Search methods that require no gradient information and can achieve a global optimal solution offer considerable advantages in solving these difficult optimization problems [13].

As a stochastic method, genetic algorithm (GA) is an emergent optimization algorithm mimicking of the natural evolution, where a biological population evolves over generations to adapt to an environment by selection, recombination, and mutation. When GA is applied to optimization problems, fitness, individual, and genes usually correspond to an objective function value, a design candidate, and design variables, respectively. In genetic algorithms, accumulated information is explored by the selection mechanism, while new regions of the search space are explored by means of genetic operators such as crossover and mutation operators. One of the key features of the genetic algorithms is that they search from multiple points in design space, instead of moving from a single point. Furthermore, these methods work on function evaluations alone and do not require derivatives or gradients of the objective function. These features lead to the advantages such as robustness, suitability to parallel computing, and simplicity. Owing to these advantages over the analytical methods, genetic algorithms have become increasingly popular in a broad class of design problems [14].

Although there are some successful applications to compute optimal solutions for aerodynamic problems, sometimes, premature or slow rate convergence may prevent GAs from reaching global optimal solution. This may be directly because of applied optimization algorithm itself or selected representation method. There are some studies [15, 16] which include global optimization methods to analysis the accuracy of some parameterizations such as B-spline and basis function approaches or Parsec method and its derivations.

In this article, two different curve representation methods; Parsec and Bezier representation methods are tested via vibrational genetic algorithm to show the effect of representation method on search type optimization process in 2-D airfoil design. At first representation methods are tested in low speed flow conditions within the inverse design problem and then the same representation methods are tested in transonic flow conditions within the optimization problem. For both cases vibrational genetic algorithm is used as an optimization tool.

From the results obtained, it is concluded that Parsec method has a better performance in subsonic flow conditions within the inverse design problem. On the other hand, it is concluded that Bezier representation method is more efficient than Parsec in transonic flow regimes.

2. METHODOLOGY

Within methodology title firstly representation methods, secondly computational fluid dynamics tools, and finally VGA as optimization tool are described.

Representation Methods

Bezier and Parsec representation methods are polynomial function based representation methods. Although there are some similarities between Bezier and Parsec methodologies these curves are different in nature.

A selected 2-D curve, an airfoil can be represented by Bezier curve representation with a set of *(m+1)* control points. Its expression is given by the following equations:

$$x(t)=\sum_{i=0}^{m} C_m^i \, t^i \,(1-t)^{m-i}\, x_i$$

$$y(t)=\sum_{i=0}^{m} C_m^i \, t^i \,(1-t)^{m-i}\, y_i \qquad (1)$$

$$C_m^i=\frac{m!}{i!\,(m-i)!}$$

where t is the parameter of the curve whose values vary uniformly between [0-1]. The (x_i, y_i) are the coordinates of the control points which define the profile coordinates $(x(t), y(t))$. The two control points (0,0) and (1,0) at the leading and trailing edge are fixed. It is commonly considered the x_i fixed, and the parameters coded in the genetic algorithm are only the y_i coordinates of the control points. Fig. 1 shows the Bezier curve representation of NACA 4-digit airfoil.

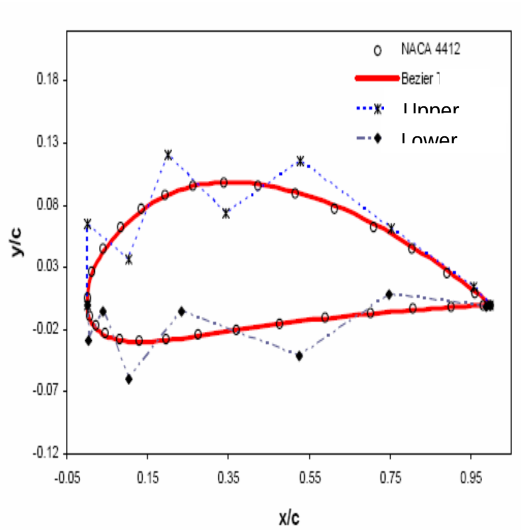

Figure 1. An airfoil in Bezier form.

An airfoil family, "Parsec", has been proposed to parameterize an airfoil shape in a different way but familiar to geometry. A remarkable point is that this technique has been developed aiming to control important aerodynamic features effectively by selecting the design parameters based on the knowledge of flows around airfoil. The Parsec-11 basic set parameterizes upper and lower airfoil surfaces using polynomials in coordinates *x, z* as

$$z=\sum_{n=1}^{6} a_n x^{n-1/2} \qquad (2)$$

where a_n are real coefficients. Instead of taking these coefficients as design parameters, the Parsec airfoils are defined by basic geometric parameters; leading edge radius, upper and lower crest location including curvatures, trailing edge ordinate, thickness, direction, and wedge angles as shown in Fig. 2.

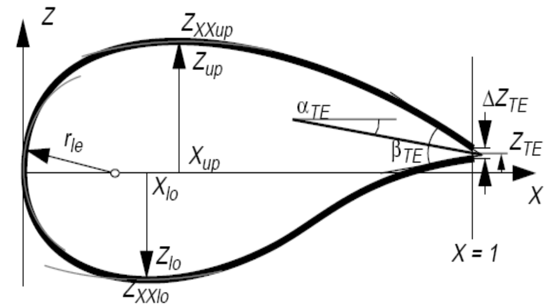

Figure 2. Design parameters for the Parsec airfoil.

The real coefficients, a_n, are computed by solving simple simultaneous equations related to each design parameter. An algebraic equation system can be written in accordance with design parameters and solved to get coefficients for each curve. For the upper surface of the airfoil, the design parameters can be related to the polynomials as

$$\begin{aligned}
z_{TE} &= z(1)=\sum_{n=1}^{6} a_n \\
z_{UP} &= z(x_{up})=\sum_{n=1}^{6} a_n x_{up}^{\,n-1/2} \\
\tan(\alpha_{TE}) &= z'(1)=\sum_{n=1}^{6}\frac{2n-1}{2}a_n \\
z'(x_{up}) &= \sum_{n=1}^{6}\frac{2n-1}{2}a_n x_{up}^{\,n-3/2} \\
z_{XXUP} &= z''(x_{up})=\sum_{n=1}^{6}\frac{2n-1}{2}\frac{2n-3}{2}a_n x_{up}^{\,n-5/2} \\
r &= \frac{1}{2}a_1^2
\end{aligned} \qquad (3)$$

The similar equations are valid for the lower surface of the airfoil. By using these equations, the following algebraic equation system can be written:

$$\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 \\
x_{up}^{1/2} & x_{up}^{3/2} & x_{up}^{5/2} & x_{up}^{7/2} & x_{up}^{9/2} & x_{up}^{11/2} \\
1/2 & 3/2 & 5/2 & 7/2 & 9/2 & 11/2 \\
\frac{1}{2}x_{up}^{-1/2} & \frac{3}{2}x_{up}^{1/2} & \frac{5}{2}x_{up}^{3/2} & \frac{7}{2}x_{up}^{5/2} & \frac{9}{2}x_{up}^{7/2} & \frac{11}{2}x_{up}^{9/2} \\
-\frac{1}{4}x_{up}^{-3/2} & \frac{3}{4}x_{up}^{-1/2} & \frac{15}{4}x_{up}^{1/2} & \frac{35}{4}x_{up}^{3/2} & \frac{63}{4}x_{up}^{5/2} & \frac{99}{4}x_{up}^{7/2} \\
1 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix}
=
\begin{bmatrix} z_{TE} \\ z_{UP} \\ \tan(\alpha_{TE}) \\ 0 \\ z_{XXUP} \\ \sqrt{2r} \end{bmatrix} \qquad (4)$$

Parsec and Bezier curve representation methods are still popular fashions although there are some criticized points related to Bezier or Parsec. In each method some improvements are implemented to basic descriptions and compensated inefficient sides of each method.

CFD Solvers

Once an aerodynamic shape representation is defined, a numerical optimization method is coupled with a suitable flow analysis tool (flow solver). Two types of CFD solvers are used in this article. These solvers are 2-D vortex-panel solver for incompressible, inviscid, subsonic flows and Euler equations solver for inviscid, compressible, transonic flows. Panel method based solver is used in inverse design problem. The other one is used in an optimization problem.

Euler equations solver program uses elliptic partial differential equation solution method to generate structural grids around the airfoil. The produced grid structure example around NACA 4-digit airfoil is given in Fig. 3. Within the program, the flux values are calculated by using a cell-centered finite volume space discretization method on a structured O-mesh and Roe flux difference splitting method. The steady state solution is reached by pseudo-time marching the Euler equations using an explicit six-stage Runge-Kutta scheme.

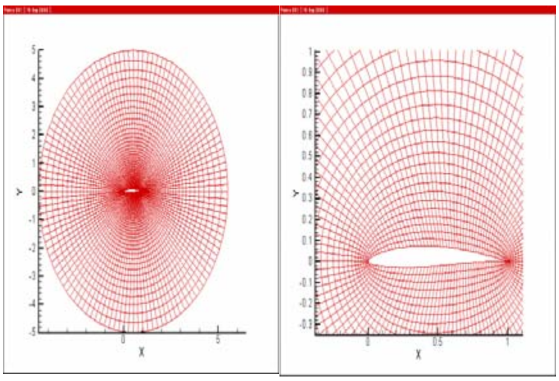

Figure 3. Grid structure for NACA 4412

Optimization Tool

As an optimization tool, VGA is a kind of real coded genetic algorithm which is mainly based on a vibration concept. It is well defined in [17] and applied in [18, 19]. In the beginning of the optimization process, GA needs a group of initial population, let's say probable solutions. First way consists of using randomly produced solutions formed by a random number generator. The second way is to create initial population from a reasonable starting individual. Initial population generation, genotype and phenotype identifications in each test case are expressed within each application. The adaptability to the environment or in other words suitability to convergence criteria is measured by fitness function value. Fitness functions are expressed in each case. After deciding on fitness value computation, the second decision to make in using a genetic algorithm is how to perform selection. The purpose of selection is, of course, to emphasize the fitter individuals in the population in hopes that their offspring will in turn have even higher fitness. In each case selection model is expressed.

The difference between VGA and regular GA is how to apply mutation operator to the individuals. At reproduction phase, in different periods applying a vibrational mutation operator to all genes of the whole population, the individuals in the population spread out through the design space. Although vibrational mutation operator causes a great mutation rate resulting in the distribution of the individuals randomly, elitism concept makes the distribution stay on the right path. Vibration strategy in mutation is applied right after the cross over phase. In the first step, entire genes in all chromosomes are mutated as follows;

$$x_i^k = x_i^k\,[1 + w.\beta.(1-u)] \qquad k = 1,2,3,\ldots,n \quad i = 1,2,3,\ldots,m \qquad (5)$$

where $x^k{}_i$ is the gene, m is the chromosome length (total gene number of a chromosome), n is the total number of individuals in the population, β is the main amplitude, u is a random real number between [0-1], and w is a user defined real weight number. It controls β. Implementation of the vibrational mutation starts from a certain gene position at first chromosome, and continues throughout the genes at the same positions in the other chromosomes. This process is applied to all individuals in the population in every f generation.

3. CASE STUDIES AND RESULTS

In the first case of first study; representation methods are tested in low speed flow conditions within the inverse design problem. In the second case; the same representation methods are tested in transonic flow conditions within the optimization problem. For both cases vibrational genetic algorithm is used as an optimization tool.

1st Case

In the first test case, initial populations for both representation methods are generated by using random number operator based on NACA 0012 symmetric airfoil. Bezier curves for upper and lover airfoil lines are governed by 26 control points. Four control points of them are known points such as leading edge and trailing edge. Therefore the total unknown number of control points for an airfoil is 22. The sample initial population for Bezier curves is given in Fig. 4.

Figure 4. Initial population based on Bezier curves.

Parsec curves are totally directed by 10 parameters. The y coordinate and thickness of trailing edge are fixed as zero. The sample initial population for Parsec curves is given in Fig. 5.

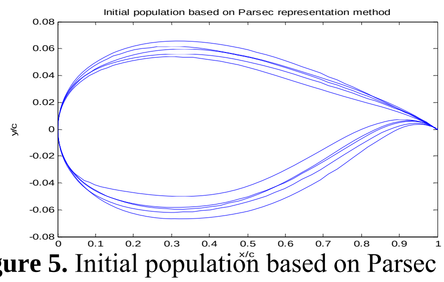

Figure 5. Initial population based on Parsec curves

For both processes angle of attack is fixed 0. The fitness function Φ_i for i_{th} individual among population is defined as

$$\Phi_i = 1/f_i \qquad f_i = \int (Cp^i - Cp^r)ds \qquad (6)$$

where Cp^i is the pressure coefficient value of i_{th} individual, Cp^r is the pressure coefficient value of the reference curve. As a reference curve *Cp* distribution around Rae 2822 asymmetric airfoil is selected. Initial / reference *Cp* distributions together with initial / target airfoil shapes are shown in Fig. 6.

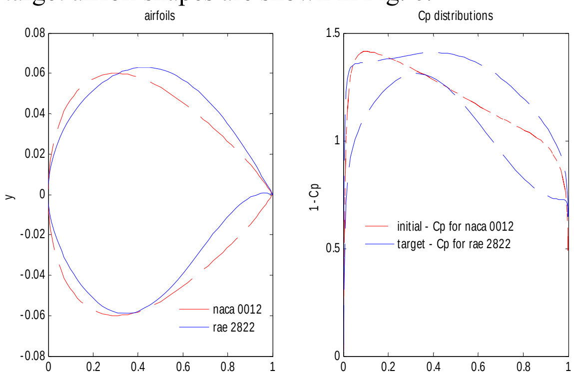

Figure 6. Initial and reference *Cp* distributions together with initial and target airfoil shapes

The features of common optimization tool, VGA, are selected as follows; maximum generation is limited to 100, population number is 10, selection method is roulette, elite count is 1, mutation is vibrational mutation operator with frequency 4 and weight 0.5, cross over is modified Blx-α with α 0.8. At the end of the optimization processes the resulted typical *Cp* distributions and airfoil shapes for both representations are depicted in Fig. 7.

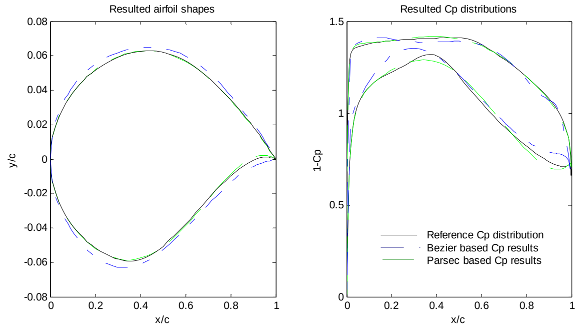

Figure 7. At the end of the optimization processes the resulted typical *Cp* distributions and airfoil shapes.

The comparison between Bezier and Parsec are shown in Fig. 8. The plot gives average best individual (over 20 runs) against generations. Fig. 8 emphasizes the superiority of Parsec representation method. Regarding the average best individual fitness value, Parsec gives better results than Bezier representation method while the second method shows more than 300% improvement in the final fitness value. The maximum, minimum and average fitness values for two methods are shown in Table I. It is clear from this table that Parsec representation method is more efficient than the other one.

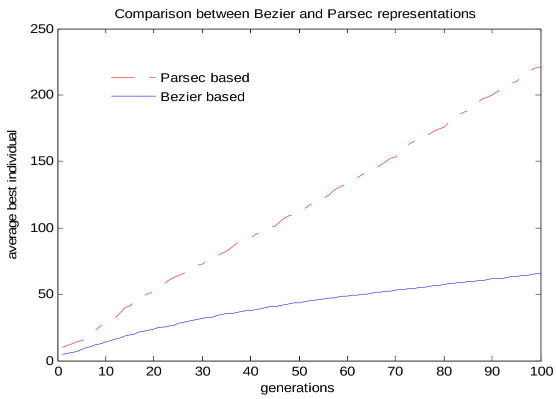

Figure 8. Comparison between Bezier and Parsec representations in accordance with average best individual

Representation method	Bezier	Parsec
Max	78	402.5
Min	48.8	150.5
Average	65.4	221.2

Table 1. Fitness values and methods

2nd Case

In the second test case shock wave reduction problem of Rae 2822 airfoil at 2° angle of attack and Mach number 0.75 was investigated via two representation methods within VGA process. Initial populations for both representation methods are generated by using random number operator based on Rae 2822 airfoil. Bezier curves are governed by 22 control points.

Parsec curves are directed by 10 parameters. Sample initial populations for Parsec and Bezier curves are given in Fig. 9.

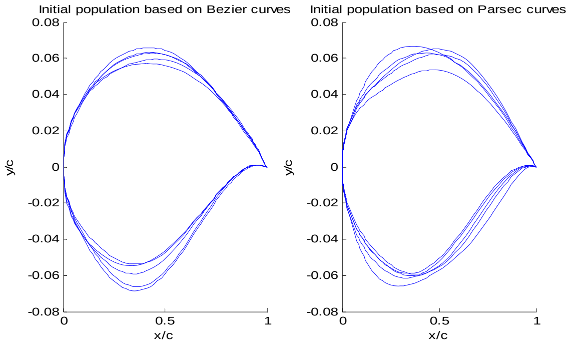

Figure 9. Initial populations based on Bezier and Parsec curves.

The fitness function, *f*, for i_{th} individual among population is defined as

$$f = \left[\frac{C_D}{C_L} + 10 \ \left(C_L^* - C_{L2}\right)^2 + 100 \ \left(t^* - t\right)^2 \right]^{-1}$$

$$C_{L2} = \begin{cases} C_L^* & if \quad C_L \geq C_L^* \\ C_L & if \quad C_L < C_L^* \end{cases} \qquad (7)$$

where C_L is the lift coefficient, C_D is the wave drag coefficient and *t* is the thickness ratio of the candidate airfoil, as C_L^* and t^* are the design lift coefficient (equal to 0.75) and thickness ratio (equal to 0.12) respectively.

The features of common optimization tool, VGA, are selected as follows; maximum generation is limited to 100, population number is 10, selection method is roulette, elite count is 1, mutation is vibrational mutation operator with frequency 4 and weight 0.5, cross over is modified Blx-α with α 0.8. The original airfoil of Rae2822 and typical optimized ones (Parsec and Bezier representation based ones, from one of 10 independent runs of VGA), and their pressure coefficient *Cp* distributions are shown in Fig. 10 correspondingly.

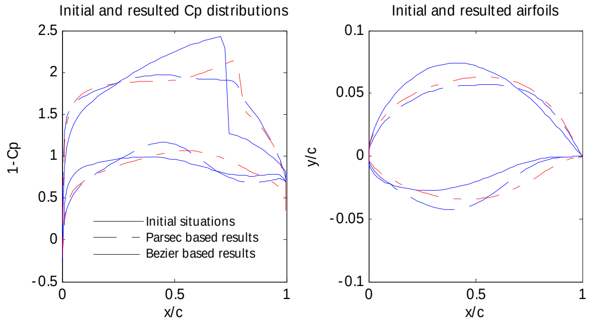

Figure 10. At the end of the optimization processes the resulted typical *Cp* distributions and airfoil shapes.

The comparison of optimization processes between Bezier and Parsec are shown in Fig. 11. The plot gives average best individual (over 10 independent runs) against generations. Fig. 11 emphasizes the superiority of Bezier representation method. Regarding the average best individual fitness value, Bezier method based optimization process gives slightly better results than Parsec method based optimization process while the first method shows more than 40% improvement in the required iteration number for cost function value 23.5.

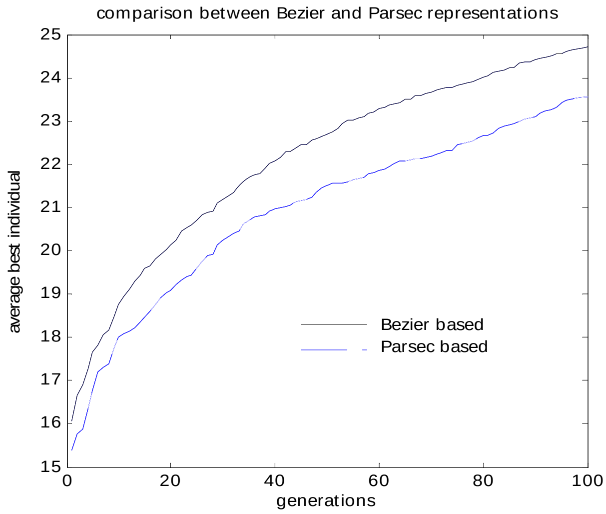

Figure 11. Comparison between Bezier and Parsec representations in accordance with average best individual.

4. CONCLUSION

In this article, Bezier and Parsec representation methods are tested in two different flow conditions; subsonic and transonic flows. In the fist test case both representation methods are compared via VGA optimization tool under the subsonic flow conditions. The comparison between Bezier and Parsec representation methods is shown in Fig. 8. This plot emphasizes the superiority of Parsec representation method. In the second test case both representation methods are compared via VGA optimization tool under the transonic flow conditions. The comparison between Bezier and Parsec representation methods is shown in Fig. 11. This plot emphasizes the superiority of Bezier representation method. From these cases it is concluded that Parsec method is more global and more efficient than Bezier method in subsonic flows. However, Bezier method is more flexible than Parsec method within transonic flows.

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VITAE

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