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THE FACTORS AFFECTING NON-LINEAR DYNAMICS OF A SPECIAL CENTRIFUGAL ADJUSTER

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ABSTRACT

The special centrifugal adjuster regulating the compression ratio of the variable compression engine [1] is a epicycle cog-wheel mechanism as demonstrated [2,3,4,5,6]. The mechanism that is actuated by centrifugal force creates relative angle shift of shafts depending on revolutions per minute (RPM). The adjuster also provides damping of vibration. The kinetical and potential energy equation has been defined by using geometrical and analytical method and consequently motion equation has been defined in accordance with II type of Lagrange equation. The differential motion equation of the adjuster has then been solved by using the finite difference method . Consequently factors affecting dynamics of the special centrifugal adjuster have been investigated and the related graphs have been plotted.

Keywords: *Special centrifugal adjuster, Variable compression engine, Finite difference method.*

ÖZEL BİR MERKEZKAÇ REGÜLATÖRÜN LİNEER OLMAYAN DİNAMİĞİNE ETKİ EDEN FAKTÖRLER

ÖZET

Değişken sıkıştırma oranlı motorların [1] sıkıştırma oranını ayarlayan özel merkezkaç regülatör, dişli çarklardan oluşmuş bir epicycle mekanizmadır [2,3,4,5,6]. Bu mekanizma merkezkaç kuvvetlerin etkisi ile harekete geçer ve devir sayına bağlı olarak şaftlarda nispi bir açısal öteleme yaratır. Regülatör aynı zamanda titreşimin sönümlenmesini de sağlamaktadır. Kinetik ve potansiyel enerji denklemleri geometrik ve analitik metotlar kullanılarak, hareket denklemi de Lagrange tip II denklemi kullanılarak çözülmüştür. Daha sonra mekanizmanın diferansiyel hareket denklemi sonlu farklar yöntemi kullanılarak çözülmüştür. Son olarak özel merkezkaç regülatörün lineer olmayan dinamiğine etki eden faktörler incelenmiş ve ilgili grafikler çizilmiştir.

Anahtar Kelimeler: *Özel merkezkaç regülatör, Değişken sıkıştırma oranlı motor, Sonlu farklar metodu.*

1. INTRODUCTION

The aim of this paper is to define the factors affecting the dynamics of an adjuster depending on RPM. The mechanism could be applied to the variable compression engines. The variable compression engines' compression ratio is regulated by the adjuster depending on RPM. There are two independent movements in the mechanism. Therefore the motion equation of the system (adjuster) will consist of two equations. For defining the motion equation of the system, firstly the kinetic and potential energy of the system (adjuster) have to be defined. The kinetic energy is defined with the transmission ratio and the potential energy is defined with the geometrical methods. Motion equation is solved by the special computer software. Both of the mechanisms are patented. Manufacturing of these mechanisms could be performed by existing machine tools. That is why the manufacturing of these mechanism are commercially beneficial.

2. PROBLEM FORMULATION

In order to define the precise dimension of the special centrifugal adjuster, it is necessary to investigate the factors affecting the non-linear dynamics of the current mechanism.

The aim of this paper is to investigate the factors affecting the non-linear dynamics of the special centrifugal adjuster in accordance with the dependence on RPM.

3. DEFINITION OF KINETIC AND POTENTIAL ENERGY OF THE SYSTEM

The kinetic energy of the system is defined as follows (Figure 1.):

$$T_{SYS} = T_1 + T_2 + T_3 , \qquad (1)$$

where, T_{SYS} is the kinetic energy of the system (adjuster), T_1, T_2, T_3 are the kinetic energy of the cog-wheels regarding the indexes:

$$\begin{cases} T_1 = \frac{1}{2} J_1 \omega_1{}^2 \\ T_2 = \sum_i \frac{1}{2} m_{p_i} v_{p_i}{}^2 \\ T_3 = \frac{1}{2} J_3 \omega_3^2 \end{cases} ; \qquad (2)$$

where, J_1, J_3 - are the inertia moments of the members regarding the indexes, ω_1, ω_3 - are the angular velocities of the cog-wheels regarding the indexes, m_{p_i} - is the weights of the elementary parts of the eccentric fixed satellite, $\vec{v}_{p_i}$ - is the velocity of the elementary parts of the eccentric fixed satellite which is defined as follows:

$$\vec{v}_{p_i} = \vec{v}_{O_2} + \vec{v}_{P_iO_2} , \qquad (3)$$

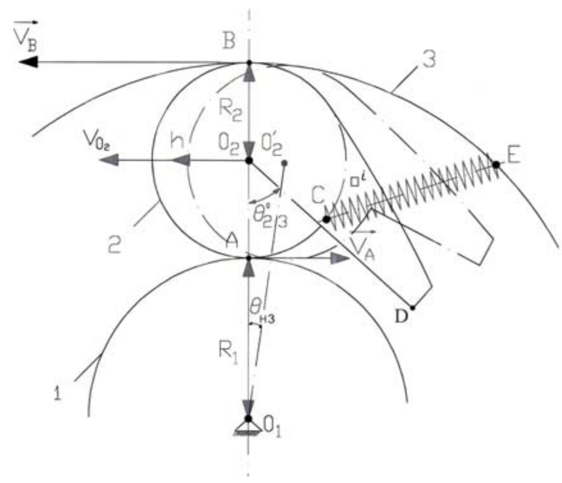

Figure 1. The kinematical scheme of the adjuster.

where, $\vec{v}_{O_2}$ - is the velocity of the O_2 point, ($v_{O_2} = \omega_H (R_1 + R_2)$), the ω_H - is the angular velocity of the point O_2 and R_1, R_2 – are the radiuses of the cog-wheels regarding the indexes; $v_{p_iO_2} = \omega_2 R_{p_i}$; $\vec{v}_{p_iO_2}$ - is the velocity of the elementary part in relation to the O_2 point, ω_2 - is the angular velocity of the eccentric fixed satellite and R_{p_i} - is the distance from arbor O_2 to the elementary part p_i; the equation of kinetic energy could be written as follow if the Eqs. (2) and (3) and aforementioned consider in the Eq. (1):

$$\begin{aligned} T_{SYS} &= \frac{1}{2} J_1 \omega_1^2 + \frac{1}{2} J_3 \omega_3^2 + \frac{1}{8} M_2 \left[\omega_3 (R_1 + 2R_2) + \omega_1 R_1\right]^2 - \\ &- \frac{1}{4R_2} M_2 r_C \left[\omega_3{}^2 (R_1 + 2R_2)^2 - \omega_1{}^2 R_1{}^2\right] \times \\ &\times \cos\left[\frac{R_1(\theta_3 - \theta_1) + 2R_2\theta_{2.3}^0}{2R_2}\right] + \\ &+ \frac{1}{8R_2^2} J_S \left[\omega_3 (R_1 + 2R_2) - \omega_1 R\right]^2 \end{aligned} \qquad (4)$$

where, M_2 - is the weight of satellite with eccentric weight, r_C - is the distance from the O_2 point to center of gravity of the satellite eccentric, $\theta_3 - \theta_1 = \Delta\theta$ - is the relative turning angle of the shafts, $\theta_{2.3}^0$ - is the angle between the line in relation to the vertical symmetrical axes which of the eccentric fixed satellite and between center of gravity, where θ_1, θ_3 - are the absolute turning angles of the cog-wheels regarding the indexes, J_S - is the inertia moment of the satellite in relation to the O_2 point.

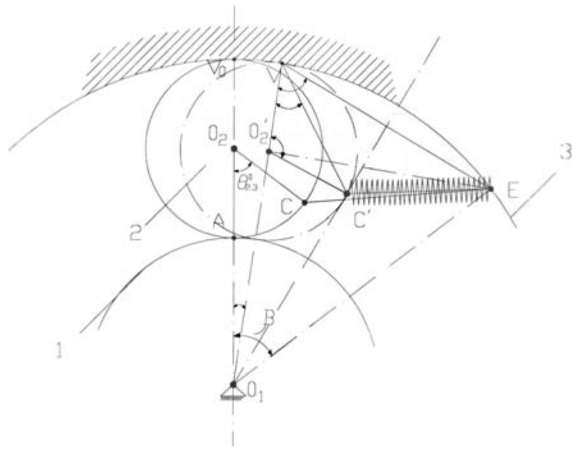

Figure 2. The geometrical scheme of the adjuster.

The potential energy of system is equal to the energy of the spring and is defined as follows (Figure2.):

$$V_{sis} = \frac{1}{2}\cdot k\cdot\left\{L_0 - \sqrt{\begin{aligned}&4R_2^2\cos^2\left[\frac{(\theta_3-\theta_1)R_1+2R_2\theta_{2.3}^0}{4R_2}\right]+\\&+4(R_1+2R_2)\sin^2\left[\frac{\beta}{2}-\frac{R_1(\theta_3-\theta_1)}{4(R_1+R_2)}\right]-\\&-8R_2(R_1+2R_2)\cdot\cos\left[\frac{(\theta_3-\theta_1)R_1+2R_2\theta_{2.3}^0}{4R_2}\right]\times\\&\times\sin\left[\frac{\beta}{2}-\frac{R_1(\theta_3-\theta_1)}{4(R_1+R_2)}\right]\times\\&\times\sin\left[\frac{\beta}{2}-\frac{R_1(\theta_3-\theta_1)}{4(R_1+R_2)}+\frac{(\theta_3-\theta_1)R_1+2R_2\theta_{2.3}^0}{4R_2}\right]\end{aligned}}\right\}^2 \quad (5)$$

where, V_{SYS} - is the potential energy of the system, k is the spring constant, L_0 - is the initial length of the spring and β - is the angle of setting of the spring.

4. DIFFERENTIAL MOTION EQUATION OF THE SPECIAL CENTRIFUGAL ADJUSTER

The differential motion equation of the special centrifugal adjuster could be written as follows if the Eqs. (4) and (5) take into account in the formula of the Lagrange equation [7,8,9,10]:

$$\begin{cases}\ddot{\theta}_1\cdot\phi_1(\theta_3-\theta_1)+f_1(\dot{\theta}_1,\dot{\theta}_3,\theta_3-\theta_1)=Q_1\\ \ddot{\theta}_3\cdot\phi_4(\theta_3-\theta_1)+f_2(\dot{\theta}_1,\dot{\theta}_3,\theta_3-\theta_1)=Q_2\end{cases} \quad (6)$$

where, $\ddot{\theta}_1$ - is the double derivative in relation to the time of angular velocity of the sun cog-wheel, $\ddot{\theta}_3$ - is the double derivative in relation to the time of angular velocity of the crown cog-wheel, $\dot{\theta}_1$ and $\dot{\theta}_3$ - are the derivatives in relation to the time of angular velocity of the cog-wheels regarding the indexes.

Where as:

$$\phi_1(\theta_3-\theta_1)=J_1+\frac{1}{4}M_2R_1^2+\frac{1}{2R_2}M_2r_CR_1^2\times$$
$$\times\cos\left[\frac{R_1(\theta_3-\theta_1)+2R_2\theta_{2.3}^0}{2R_2}\right]+\frac{J_SR_1^2}{4R_2^2}$$

$$\phi_4(\theta_3-\theta_1)=J_3+\frac{1}{4}M_2(R_1+2R_2)^2-$$
$$-\frac{1}{2R_2}M_2r_C(R_1+2R_2)^2\cos\left[\frac{R_1(\theta_3-\theta_1)+2R_2\theta_{2.3}^0}{2R_2}\right]+$$
$$+\frac{1}{4R_2^2}J_S(R_1+2R_2)^2$$

$$f_1(\dot{\theta}_1,\dot{\theta}_3,\theta_3-\theta_1)=A_1+B-\xi(\Gamma+\mathrm{E}-\mathrm{Z}-\mathrm{H}+\mathrm{N})$$

$$f_2(\dot{\theta}_1,\dot{\theta}_3,\theta_3-\theta_1)=A_2-B-\xi(-\Gamma-\mathrm{E}+\mathrm{Z}+\mathrm{H}-\mathrm{N})$$

$$A_1=-\frac{R_1}{4R_2^2}M_2r_CR_1^2\times$$
$$\times\sin\left[\frac{R_1(\theta_3-\theta_1)+2\theta_{2.3}^0R_2}{2R_2}\right]\cdot(\dot{\theta}_3-\dot{\theta}_1)$$

$$\mathrm{B}=\frac{R_1}{8R_2^2}M_2r_C\left[(R_1+2R_2)^2\dot{\theta}_3^2-R_1^2\dot{\theta}_1^2\right]\times$$
$$\times\sin\left[\frac{R_1(\theta_3-\theta_1)+2\theta_{2.3}^0R_2}{2R_2}\right]$$

$$A_2=\frac{R_1}{4R_2^2}M_2r_C(R_1+2R_2)^2\times$$
$$\times\sin\left[\frac{R_1(\theta_3-\theta_1)+2\theta_{2.3}^0R_2}{2R_2}\right]\cdot(\dot{\theta}_3-\dot{\theta}_1)$$

$$\xi=k\Delta L\frac{1}{2L}$$

$$\Gamma=2R_2\cos\left[\frac{R_1(\theta_3-\theta_1)+2\theta_{2.3}^0R_2}{4R_2}\right]\times$$
$$\times\sin\left[\frac{R_1(\theta_3-\theta_1)+2\theta_{2.3}^0R_2}{4R_2}\right]R_1$$

$$\mathrm{E}=\frac{1}{2}\left(R_1+2R_2\right)^2\sin\left[\frac{\beta}{2}-\frac{R_1\left(\theta_3-\theta_1\right)}{4\left(R_1+R_2\right)}\right]\times$$
$$\times\cos\left[\frac{\beta}{2}-\frac{R_1\left(\theta_3-\theta_1\right)}{4\left(R_1+R_2\right)}\right]\frac{R_1}{R_1+R_2}$$

$$\mathrm{Z}=2\left(R_1+2R_2\right)\sin\left[\frac{R_1\left(\theta_3-\theta_1\right)+2\theta_{2.3}^0R_2}{4R_2}\right]R_1\times$$
$$\times\sin\left[\frac{\beta}{2}-\frac{R_1\left(\theta_3-\theta_1\right)}{4\left(R_1+R_2\right)}\right]\times$$
$$\times\sin\left[\frac{\beta}{2}-\frac{R_1\left(\theta_3-\theta_1\right)}{4\left(R_1+R_2\right)}+\frac{R_1\left(\theta_3-\theta_1\right)+2\theta_{2.3}^0R_2}{4R_2}\right]$$

$$\mathrm{H}=2R_2\left(R_1+2R_2\right)\cos\left[\frac{R_1\left(\theta_3-\theta_1\right)+2\theta_{2.3}^0R_2}{4R_2}\right]\times$$
$$\times\cos\left[\frac{\beta}{2}-\frac{R_1\left(\theta_3-\theta_1\right)}{4\left(R_1+R_2\right)}\right]\frac{R_1}{R_1+R_2}\times$$
$$\times\sin\left[\frac{\beta}{2}-\frac{R_1\left(\theta_3-\theta_1\right)}{4\left(R_1+R_2\right)}+\frac{R_1\left(\theta_3-\theta_1\right)+2\theta_{2.3}^0R_2}{4R_2}\right]$$

$$\mathrm{N}=2R_2\left(R_1+2R_2\right)\cos\left[\frac{R_1\left(\theta_3-\theta_1\right)+2\theta_{2.3}^0R_2}{4R_2}\right]\times$$
$$\times\cos\left[\frac{\beta}{2}-\frac{R_1\left(\theta_3-\theta_1\right)}{4\left(R_1+R_2\right)}\right]\times$$
$$\times\cos\left[\frac{\beta}{2}-\frac{R_1\left(\theta_3-\theta_1\right)}{4\left(R_1+R_2\right)}+\frac{R_1\left(\theta_3-\theta_1\right)+2\theta_{2.3}^0R_2}{4R_2}\right]\times$$
$$\times\frac{R_1^2}{\left(R_1+R_2\right)R_2}$$

$$Q_1=M_K\left[\frac{\alpha_1}{1+\dot{\theta}_1}+\left(\gamma_2+\frac{\gamma_1}{\dot{\theta}_1}\right)\sin\left(\dot{\theta}_1\cdot t\right)\right]$$

$$Q_2=M_K\left[\frac{\alpha_1}{1+\dot{\theta}_3}+\gamma_2\sin\left(\dot{\theta}_3\cdot t\right)\right]$$

where, Q_1 and Q_2 - are the general external oscillatory forces applied in the engine, which in practically could affect the mechanism, $\alpha_1,\alpha_2,\gamma_1,\gamma_2,M_K$ - are the constants which characterize the force and t - is time.

5. SOLVING OF DIFFERENCIAL MOTION EQUATION

The Eq. (6) system of equation can be written as follows if the derivatives substitute for the finite differences as demonstrated [11]:

$$\begin{cases}\theta_{1_{i+1}}=\dfrac{Q_{1i}-f_{1i}\left(\dot{\theta}_1,\dot{\theta}_3,\theta_3-\theta_1\right)}{\phi_{1i}\left(\theta_3-\theta_1\right)}\cdot\Delta t^2+2\theta_{1_i}-\theta_{1_{i-1}}\\ \theta_{3_{i+1}}=\dfrac{Q_{2i}-f_{2i}\left(\dot{\theta}_1,\dot{\theta}_3,\theta_3-\theta_1\right)}{\phi_{4i}\left(\theta_3-\theta_1\right)}\cdot\Delta t^2+2\theta_{3_i}-\theta_{3_{i-1}}\end{cases}\qquad(7)$$

The Eq. (7) could be solved by using the appropriate software (Microsoft Visual FoxPro, MathCAD 14 etc.) if the system of equation Eq. (6) write as an algebraically equation. So, in accordance with Cauchy problem by giving initial values to the absolute turning angles (θ_1 and θ_3), the relative turning angle ($\Delta\theta$) and the absolute spring rate and obtained as a satisfactory result.

The value of the parameters of the regulator is taken as: $R_1=a$; $R_2=1.2a$; $M_2=b$; $M_1=3.3b$; $M_3=6.6b$; $\beta=54^0$, $k=c$ and as the initial value of absolute turning angles (θ_1,θ_3) are given $\theta_{1_0}=0$, $\theta_{1_1}=-0.5$, $\theta_{3_0}=0$; $\theta_{3_1}=-0.1$. Size of unit could be taken in accordance with the dimension and with the influence force of the engine or in any mechanism the special centrifugal adjuster will be applied.

6. THE FACTORS AFFECTING NON-LINEAR DYNAMICS

The factors affecting non-linear dynamics of the adjuster are spring rate, the inertia moment of the satellite and the engine moment. For defining the influence of these factors, several values have to be given to them. In order to investigate the factors that have influence on the non-linear dynamics of the adjuster, several initial values to the spring rate, the inertia moment of the satellite and the engine moment are given. The other parameters' values are remained constant. As a result, the dependence graphs are plotted. The dependence graphs of the absolute turning angles (θ_1 and θ_3) versus the relative turning angle ($\Delta\theta$) will be as follows:

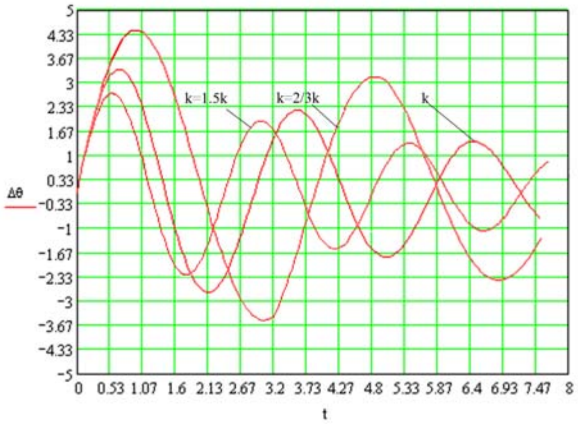

Figure 3. The time dependency of the relative angles in accordance with spring rate.

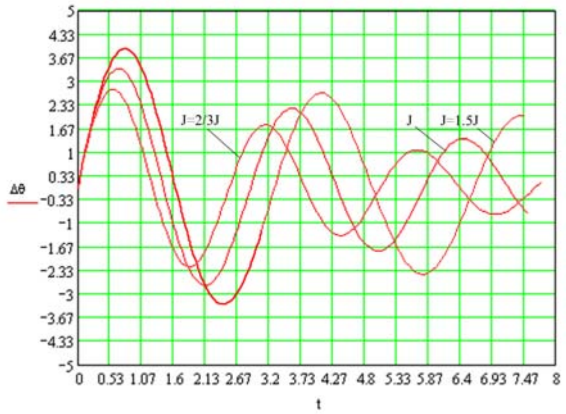

Figure 4. The time dependency of the relative angles in accordance with the inertia moment of the satellite.

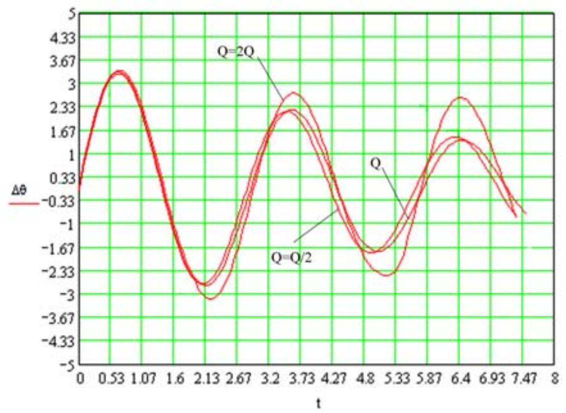

Figure 5. The time dependency of the relative angles in accordance with the engine moment.

7. CONCLUSION

In this article the finite difference method is used for solving the differential motion equation of the system. The appropriate graphs have been plotted which characterize the dynamics of system. It seems from the Figure 3, Figure 4 and Figure 5 that the fluctuation is harmonically damping oscillation. It is possible to plot other graphs by giving different initial values to the absolute turning angles (θ_1 and θ_3). It could be interpreted from the graphs that the amplitude and period of harmonically damping oscillation increases in accordance with decreasing of the spring rate and increasing of the inertia moment of the satellite and the engine moment. The amplitude and the period of harmonically damping oscillation decreases in accordance with increasing of the spring rate and decreasing of the inertia moment of the satellite and the engine moment.

The frequency of the harmonically damping oscillation increases in accordance with increasing of the spring rate and decreasing of the inertia moment of the satellite and the engine moment. The frequency of the harmonically damping oscillation increases in accordance with increasing of the spring rate and decreasing of the inertia moment of the satellite and the engine moment.

It is necessary to apply damping if the amplitude of oscillation is out the margin. It has been taken into account on construction. The oscillation damping is performed by the damping liquid or by applying the opposite spring.

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VITAE

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He worked as Engineer then auditor in the “SW Helicopter Services” LLC.

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