METU JFA 1991
(11: 1-2) 57-71

ANALYSIS OF THE GEOMETRY OF STALACTITES: BURUCIYE MEDRESE IN SIVAS

Alpay ÖZDURAL

INTRODUCTION

Received : 22. 2. 1993
Keywords: Architectural History, Buruciye Medrese in Sivas, Stalactites, History of Mathematics.

Gıyaseddin Jemshid el-Kashi, an outstanding mathematician-astronomer who lived in early 15th century, devoted a section of his book, *Miftah el-Hisab*, to stalactites. There he classifies the stalactites into four types, explains the repeating elements of each type in a systematic manner, provides some convenient rules for the survey of the areas of stalactites (without giving the explanatory proofs and calculations) and describes how to prepare the pattern for a unit element (Özdural, 1990, 37-44). One of his remarks is very enlightening concerning the underlying principles of the geometry of stalactite designs:

> As has been related, the length at the base of the largest side is called the 'scale of the stalactite' *(mikyas el-mukarnas)* (Özdural, 1990, 37).

Unfortunately for the 20th century architectural historians, el-Kashi does not give more information on the composition and design of stalactites because, being a mathematician himself and writing for surveyors, he is solely concerned with the area survey of stalactites. Nevertheless, his calculations, which are all based on the 'scale of the stalactite', easily lead us to safely assume that stalactites were designed according to a modular system based on a certain unit called the 'scale'.

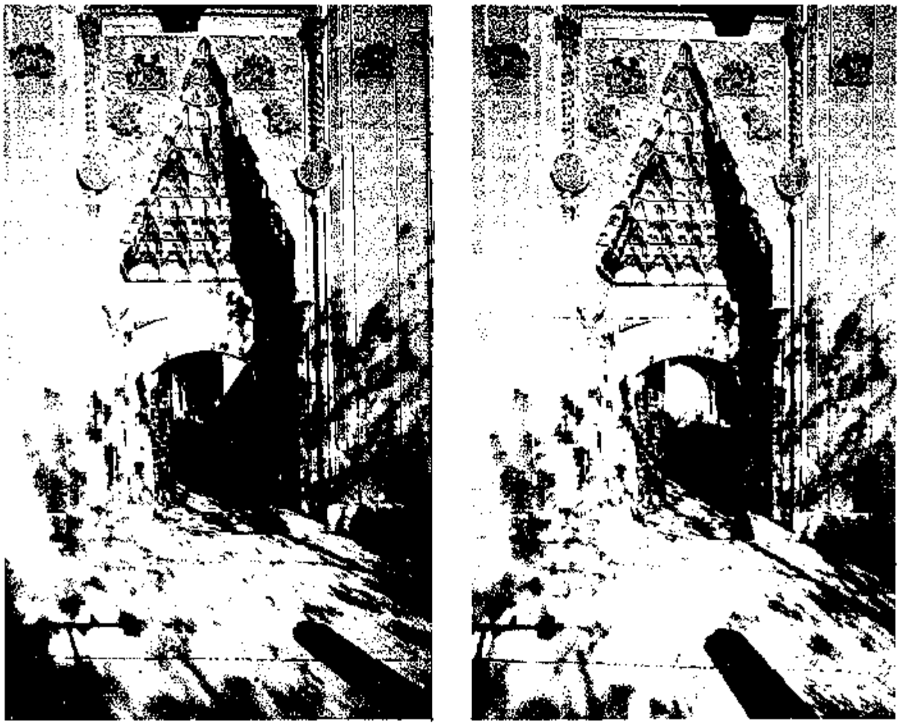

Figure 1. Stereopicture of the entrance portal of Buruciye Medrese in Sivas (METU, Architectural Photogrammetry Center Archive). The stereopair is arranged so as to give a stereoscopic image at reading distance.

El-Kashi's account was directly related to stalactites in Persia, Khurasan, Transoxiana and particularly to Samarkand. In those areas, stalactites are mainly constructed out of gypsum which, by the nature of material, depend on repeating, standardized and moulded elements. A modular design system is certainly the most natural outcome. In neighbouring countries like Anatolia, Syria and Egypt, however, the main construction material for stalactites is stone which does not necessarily require every element to be carved in separate pieces. Is the modular system of el-Kashi also applicable to stone stalactites in other regions? To answer this question, we will start by analysing the stalactite vault in the portal of Buruciye Medrese in Sivas, which is chosen arbitrarily.

BURUCIYE MEDRESE IN SIVAS

This monument is one of the finest examples of Seljukid medreses. It has four eyvans arranged in a cross-axial pattern having a central open courtyard (1). The subject of the present article, the stalactite vault, takes place over the entrance niche of the central portal which is highly decorated (**Figure 1**). Except the tile-and-brick decoration in the tomb, the whole building is constructed out of cut stone. Owing to an elaborate system of symmetry that governs the whole design from plan arrangement to smaller details, Buruciye Medrese attains a prominent position in Seljukid architecture. Unfortunately, however, some of the original details and proportions are lost as a result of a heavy restoration in 1960's.

1. For more detailed information about the monument, see Kuran (1969, 90-92), Sözen (1970, 49-57) and Gabriel (1934, 152-155).

According to the foundation inscription over the portal, the name of the founder is Muzaffer ibn Ibadullah Barujurdi and the date of construction is 1271 (H. 670). Barujurd is a town near Hamadan in Persia and, apparently, the name of the building is attributed to its founder. The signature of the founder is repeated at a position which is typically allocated for the name of the architect in Seljukid architecture. This unusual fact leads Berchem and Ethem (1917, 27) to pose the question whether the architect and the founder were the same person. This

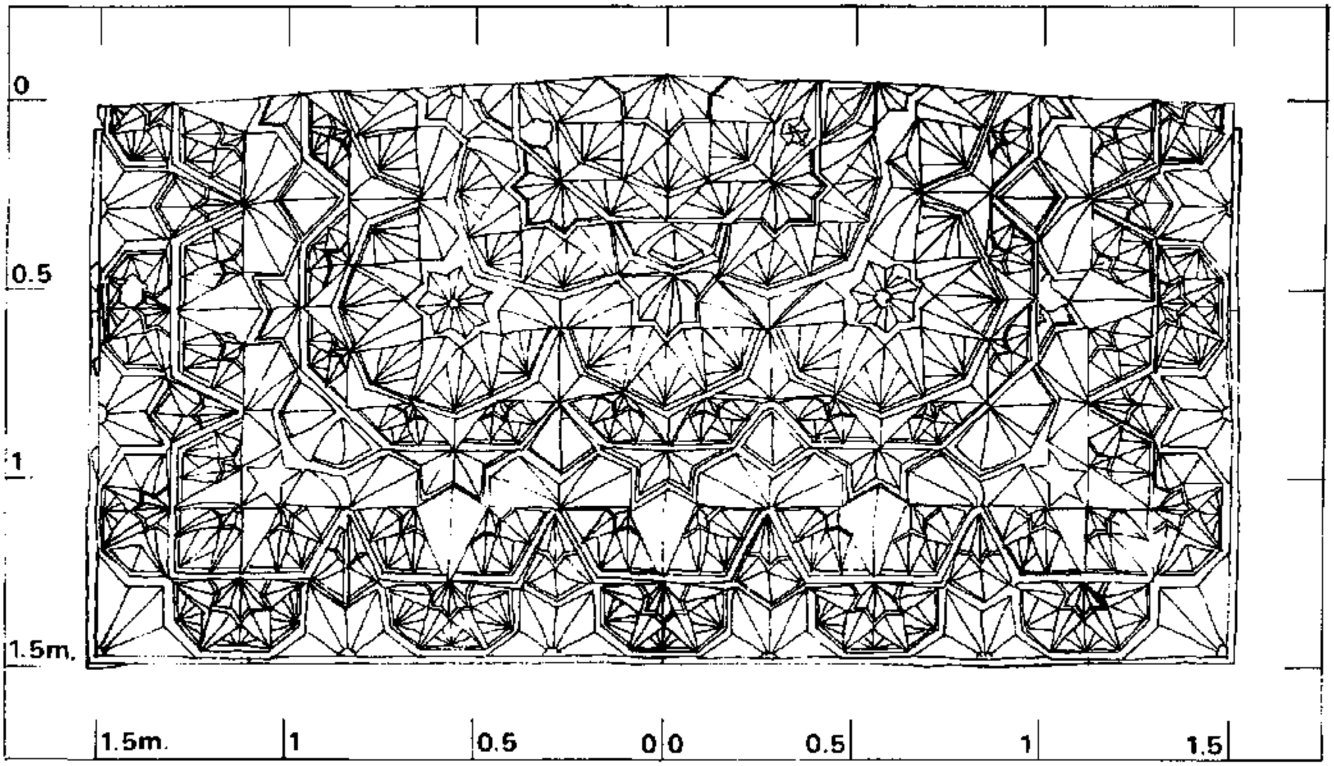

Figure 2. Photogrammetrically surveyed plan.

possibility seems more plausible when we consider the following facts: contrary to customary practice, the founder does not carry any official title; on the inscription panel in the tomb, he describes himself as 'a humble servant, helpless stranger'; the rich tile-and-brick decoration of the tomb displays similar characteristics to other examples created by Persian rather than Anatolian craftsmen.

PHOTOGRAMMETRIC SURVEY

Surveying is the preliminary, and a very crucial, stage in the analysis of stalactites. In order to obtain scrupulous results, the measured drawing should be correct and precise. Surveying has always been the most difficult part of the research. Wilber (1969, 73) expresses his experience by the following words:

> The field worker is today faced with an exhausting task merely to draw an accurate reflected plan of the stalactite system of a dome or a half-dome. After long examination from a prone position, the pattern of the system begins to appear. However, this geometry was frequently needlessly complicated.

Classical surveying techniques prove to be inefficient, complicated, arduous and it is almost impossible to obtain correct results when applied to the intricate spatial geometry of stalactites. Since the surveyor relies basically on his/her observations, the measured drawing does not usually reflect the recording of the existing pattern but actually is a restitution drawing of an idealized geometric system. This inevitably leads to certain misinterpretations because the geometry that was applied by the builders is not yet fully understood. Photogrammetric technique, on the other hand, provides the most satisfactory solution for such difficulties. It is not only the most facile and the most accurate surveying technique, but also gives the chance to produce any required drawing whenever the need arises.

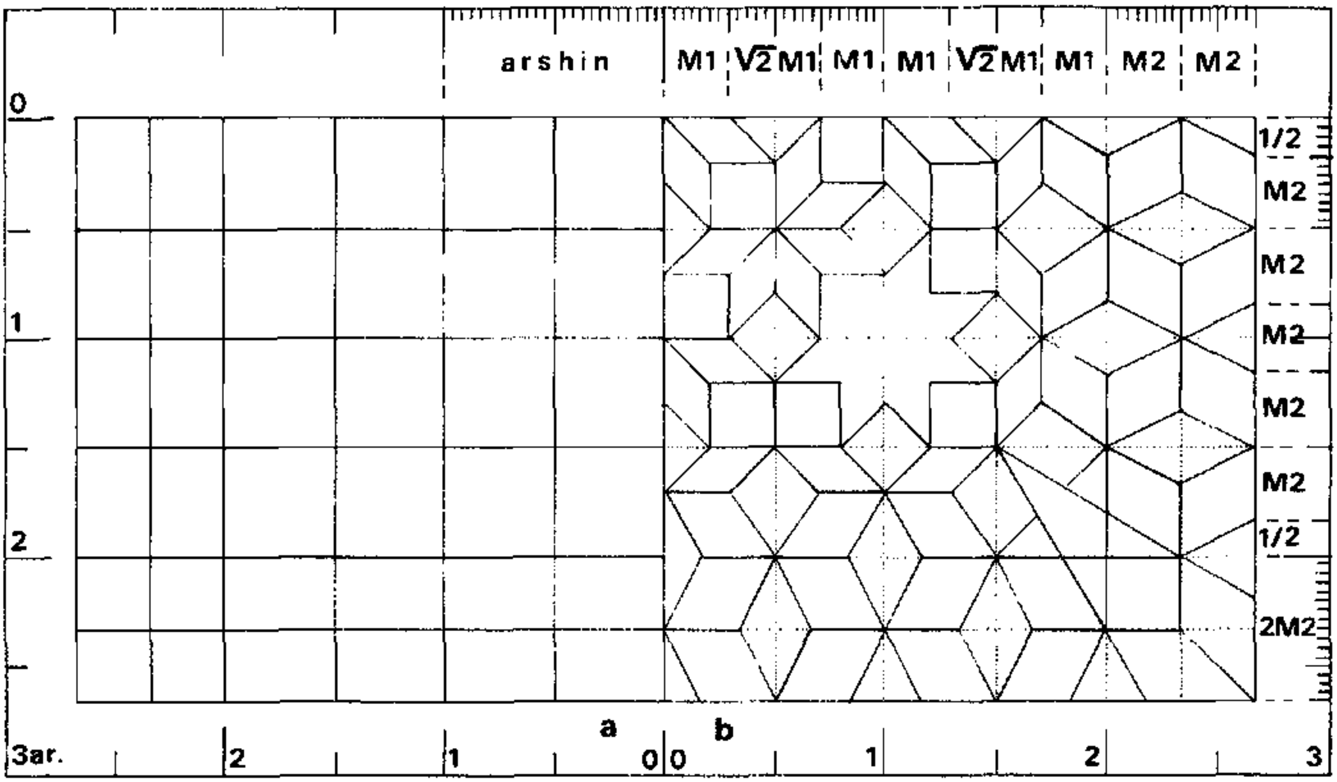

Figure 3a. Grid system based on 'arshin'.

Figure 3b. Schematic plan drawing.

In the photogrammetric survey of the Buruciye stalactite (**Figure 2**), inaccuracies which were inherent in the construction technology, aging of the material and deformations due to settlement of the building are all recorded precisely, *e.g.* the triangular outline of the exterior span of the stalactite is the result of the outward leaning of the portal (2).

PLAN DRAWING

As the first step of analysis, we note that all half-hexagrams at the lowest course are precisely equidistant from each other (56 cm) and each one corresponds to single stone pieces. Considering the construction technology of the period, we believe it is only natural to assume that 56 cm was the length of the particular measuring unit, namely *arshin* (cubit), used in this building. This assumption is the basis of our further analysis which will prove its validity and, instead of metric scale, we will use *arshin* scale henceforth (3).

Like all measuring units that were used for construction and surveying purposes in Islamic countries, this particular *arshin* was composed of 24 digits (4). When the deformations caused by aging and weathering of the building are carefully corrected, general outlines of the stalactite courses fit precisely into a grid system of *arshin* scale (**Figure 3a**). The plan of the whole stalactite is composed of a double square and the side of one square is equal to 2+2/3 *arshins*, or 64 digits.

We do not claim that the stalactite plan of Buruciye was drawn according to a grid system but we simply indicate that the particular *arshin* scale is applicable to the overall scheme. The only early stalactite plan drawing, on a plaster slab, was discovered during the excavations of the palace of Abaga Khan at Takht-i Sulaiman (c. 1275) in Azerbaijan (Harb, 1978, Pl.1). It does not show any grid pattern but displays a network of squares and rhombi, which is basically an illustration of el-Kashi's account on stalactites. Geographical and chronological proximity makes it easier to accept the same model for Buruciye (**Figure 3b**).

2. Buruciye Medrese in Sivas was surveyed photogrammetrically in 1981. Stereoscopic pictures are taken with two types of stereocameras, SMK 120 and SMK 40, on glass plates. The plotting was done on Terragraph.

We are sincerely grateful to Mr. Şinasi Kılıç for his help during the whole process of surveying.

3. Since the building is located in Anatolia, we preferred to use the Turkish word 'arshin' instead of the English equivalent, 'cubit'. We must note, however, that originally the Persian 'gez' or Arabic 'zira' could also have been used, depending upon the language of the masons of the construction.

4. This information is gathered from the author's so far unpublished research on the measuring units in the Islamic countries. 'Digits', the generic word derived from Latin, is used throughout the text because it corresponds to both 'fingers' and 'inches'. The original term could have been 'parmak' or 'bogun' in Turkish, 'engusht' or 'bend-i engusht' in Persian, 'isba' or 'burjume' in Arabic (Gökyay, 1976, 179).

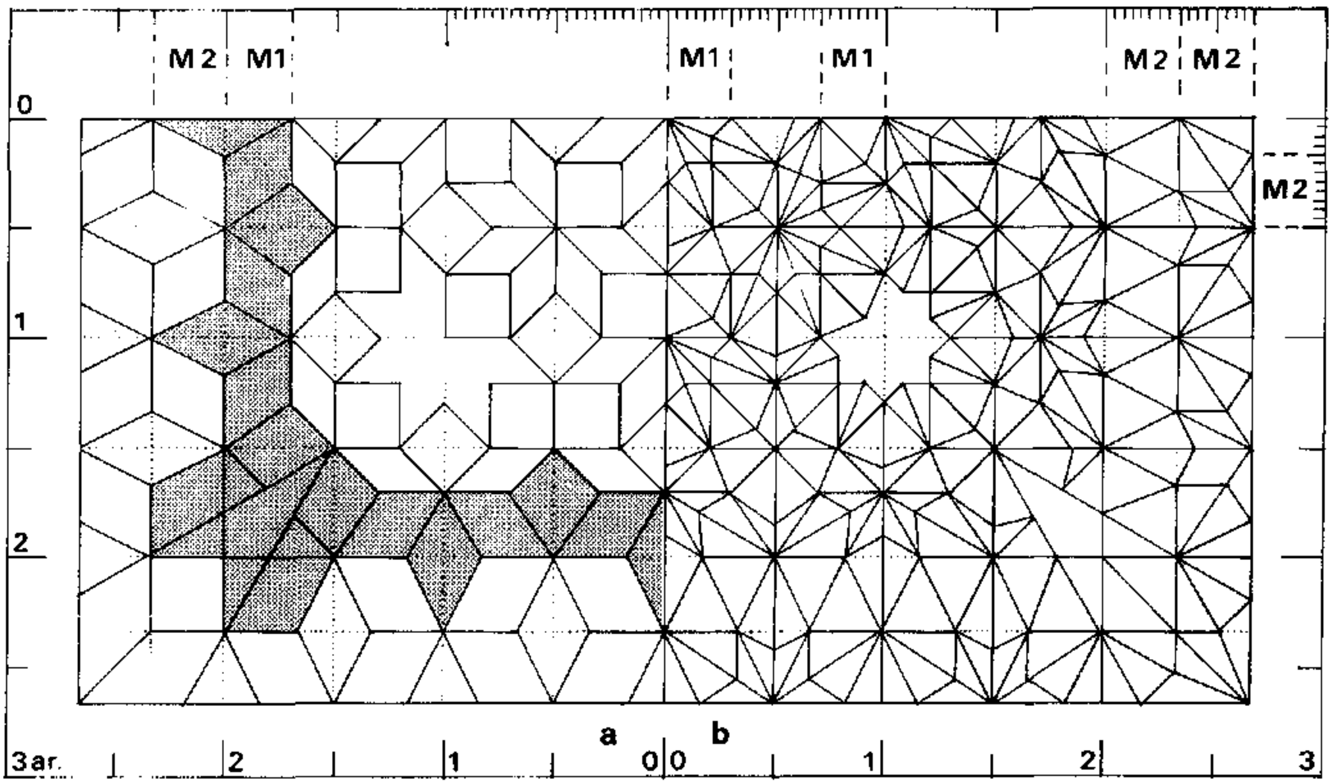

Figure 4a. Schematic plan drawing.

Figure 4b. Plan drawing showing stalactite elements.

In reality, this schematic drawing (**Figure 4a**) *per se*, was not sufficient for masons since different versions could have been produced from the same drawing. They either needed constant guidance from the designer (possibly, he was the architect) of the stalactite, or a more detailed plan showing all the actual stalactite elements (**Figure 4b**). Such detailed stalactite plans, in fact, exist in Morocco (Paccard, 1983, 303-10), in Turkey (T.S.M.K., H.1956), and in Uzbekistan (Pugachenkova, 1962, 209). They are all drawn on paper and date after the sixteenth century.

The line quality of the drawing from Takht-i Sulaiman points to the use of set squares. A 10th century mathematician, Abu'l Wafa el-Buzjani, (Ayasofya K. 2753, 7) informs us that in his time craftsmen were using *konye* (set-square) for drawing lines (**5**). An anonymous Persian craftsman- geometrician, possibly from the twelfth century, was actually using *konyes* for 30° and 36° angles (**6**). It can safely be assumed that the hypothetical plan of the stalactite of Buruciye was drawn by the aid of set-squares.

The stalactite plan of Buruciye is composed of three zones. The central zone is based on 45° angles, creating a system of $(2)^{1/2}$ relations; the peripheral zone is based on an angle of which the tangent is equal to 1:2, *i.e.* 26.565°; the transition zone integrates the above mentioned systems and is based on 30° angles, creating a system of $(3)^{1/2}$ relations (**Figures 4a, 3b**). The ratios $(2)^{1/2}$ and $(3)^{1/2}$ are irrational. They can easily be constructed by employing certain set-squares. But, in order to trace the outline plan for each course at 1:1 scale, masons need to know the dimensions of each element in advance which requires a more convenient method than using set-squares in the construction site. This is where the 'scale of the stalactite' comes into use.

El-Kashi remarks that a certain number of scales can be divided by the *arshin*, and the quotient is expressed in terms of the scale and its parts (Özdural, 1990, 39). Therefore, we can understand that the 'scale', *i.e.* the module, is a unit of length which is part of the *arshin* and it is expressed in terms of its smallest part, *i.e.*

5. He is one of the greatest Muslim mathematicians, born in Buzajan, Horasan, in 940 and died in 998 in Baghdad. He has written several books on mathematics and astronomy and commentaries to Euclid, Diaphantus, el-Khwarizmi. His chief contribution was in the development of trigonometry. His geometrical constructions were partly based on Indian models (Suter, 1960, 159).

6. The Persian treatise of this anonymous author, 'On Interlocking Similar and Corresponding Figures', is in Bibliotheque Nationale in Paris, attached to the Persian translation of Buzjani's work (Persian Ms. No 169, Folios, 180-199). Bulatov published the Russian translation of this treatise and dates it to 11th - 12th centuries (Bulatov, 1978, 325-54).

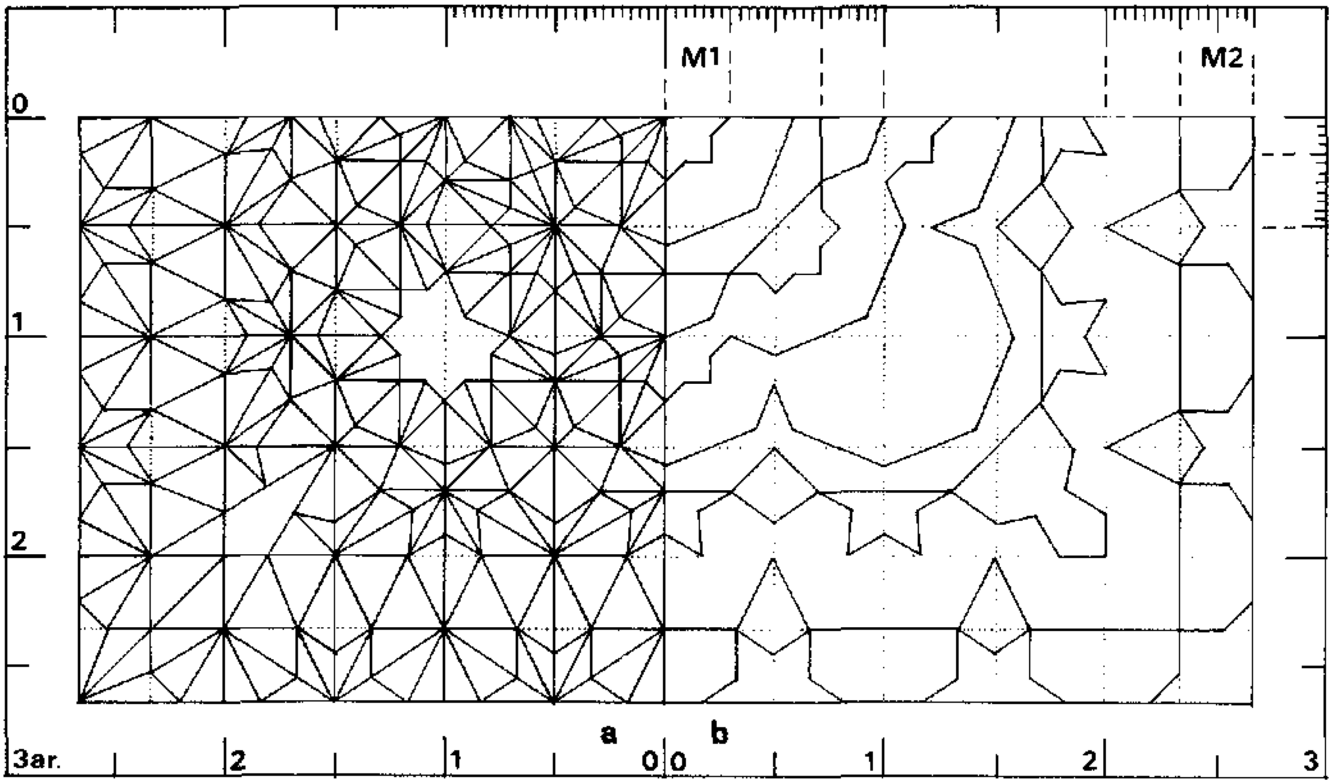

Figure 5a. Plan drawing showing stalactite elements.

Figure 5b. Outline of successive stalactite courses.

digit. By appropriate choice of the number of digits, certain irrational ratios can be expressed by the aid of certain approximation series. For example, each member of the series, 7:5, 17:12, 41:29, 99:70..., represents $(2)^{1/2}$ approximately and approaches the theoretical value as it gets higher in the series. The same is true for $(3)^{1/2}$ in the series, 7:4, 19:11, 26:15, 71:41...

A brief excursion in the history of mathematics will facilitate to appreciate the significance of these approximate ratios in history. Neugebauer (1969, 3) remarks that:

> For the history of mathematics the traditional division of political history into Antiquity and Middle Ages is of no significance...ancient methods prevailed until Newton.

Several mathematical tablets dating from Old-Babylonian times show that 7:5 and 17:12 were commonly used as approximate ratios of $(2)^{1/2}$. A recently discovered tablet reveals that they were also able to calculate the same ratio with a remarkable accuracy, 577:408=1.414213.. (Neugebauer, 1969, 35). This ratio is indeed the eleventh member of the above mentioned approximation series. The same ratio was still used by Ptolemy almost two thousand years later and was known, besides 7:5 and 17:12, to Indians as early as the fourth century B.C. (Thibaut, 1875, 227). Mesopotamians were also familiar with the $(3)^{1/2}$ series since they used 7:4 in the calculations of the equilateral triangle (Neugebauer, 1969, 47).

The discovery of irrationals marks the beginning of scientific geometry in Greek mathematics. With this new trend Greek mathematics gained a strong impetus and reached its climax in *The Elements* of Euclid. Consequently, the main emphasis of mathematics shifted to a theoretical level and axiomatic geometry attracted all the intellectual attention of future generations. The oriental tradition, however, survived in practical fields without receiving any attention. Plato acknowledges the existence of this tradition by hinting at 7:5 as the rational expression of $(2)^{1/2}$ in *The Republic* (Lindsay, 1976, 241). Indeed:

> A close study of Greek mathematics seems to give evidence that beneath the geometrical veneer there was more concern for logistic and numerical approximations than the surviving classical treatises portray (Boyer, 1968, 129).

Mathematics of Hellenistic and Roman periods can be seen as a link in an unbroken tradition from Antiquity to the Middle Ages. Heron of Alexandria (c. 75 A.D.), through his numerous works, represents this tradition. He regularly uses several of the above mentioned approximate ratios with varying degrees of accuracy (Bruins, 1964). His examples are repeated endlessly in the Roman era and during the Middle Ages in Europe (Waerden, 1954, 276). Heron's works were also translated into Arabic and were the main source behind the Islamic treatises on surveying (Schirmer, 1936, 518). El-Khwarizmi, in his famous *Algebra*, borrows heavily from him (Neugebauer, 1969, 146). Even in later times Islamic mathematicians, when confronted with irrational ratios, used rational approximations and there was a tendency in Seljukid times to think of these ratios as numbers (Juschkewitsch, 1960, 132).

These approximate ratios were generally used to express irrational ratios in terms of known dimensions. In the context of the inaccurate nature of building technology in history, these ratios provide a varying degree of accuracy which would have satified any need.

The stalactite of Buruciye is observed to be based on two modules creating two sets of modular systems and a transition zone which brings them together **(Figures 5, 4, 3)**. M1 is equal to 7 digits and is the module for the central zone. Other dimensions are expressed by the approximate ratios of $(2)^{1/2}$, taking M1 as the basis:- $10{:}7=(2)^{1/2}=7{:}5$. This system creates an approximate isosceles right-angled triangle: $2x5^2=50; 7^2=49$. M2 is equal to 8 digits and is the module for the peripheral zone which is based on the ratio of 1:2. This system creates an approximate right-angled triangle: $4^2+8^2=80; 9^2=81$. In the transition zone, $(3)^{1/2}$is expressed by the ratio of 7:4 which creates an approximate right-angled triangle: $4^2+7^2=65; 8^2=64$. This 4-7-8 triangle is the common link between the two modular systems because it includes both modules. The lowest identifiable ratios of each series are chosen simply because larger modules would be unmanageable.

The central zone of the stalactite of Buruciye **(Figure 5a)** illustrates a certain composition of stalactite elements mentioned by el-Kashi (Özdural, 1990) **(Figure 3)**. Squares are always divided into complementary forms (two-long-leggeds and almonds). Star formations are composed of complementary forms which divide a rhombus (two-short-leggeds and almonds). Undivided rhombi surround the star formations. Barleycorns are not used because the uppermost course is simply a repetition of other stars (Özdural, 1990, N.11). Same rules apply to the transition and the peripheral zones, except that the elements are based on different geometrical systems. Once the plan is ready, the next stage is to prepare the courses according to it **(Figure 5b)**.

PREPARATION OF STONE COURSES

In contemporary stalactite constructions in Iraq and Morocco a common technique is used. Gypsum or wooden form strips are prepared corresponding to the outline of each course and they are horizontally embedded within the growing system of stalactites (Wilber, 1969, 73). Spiers (1888, 45) records a similar technique in 19th century Persia:

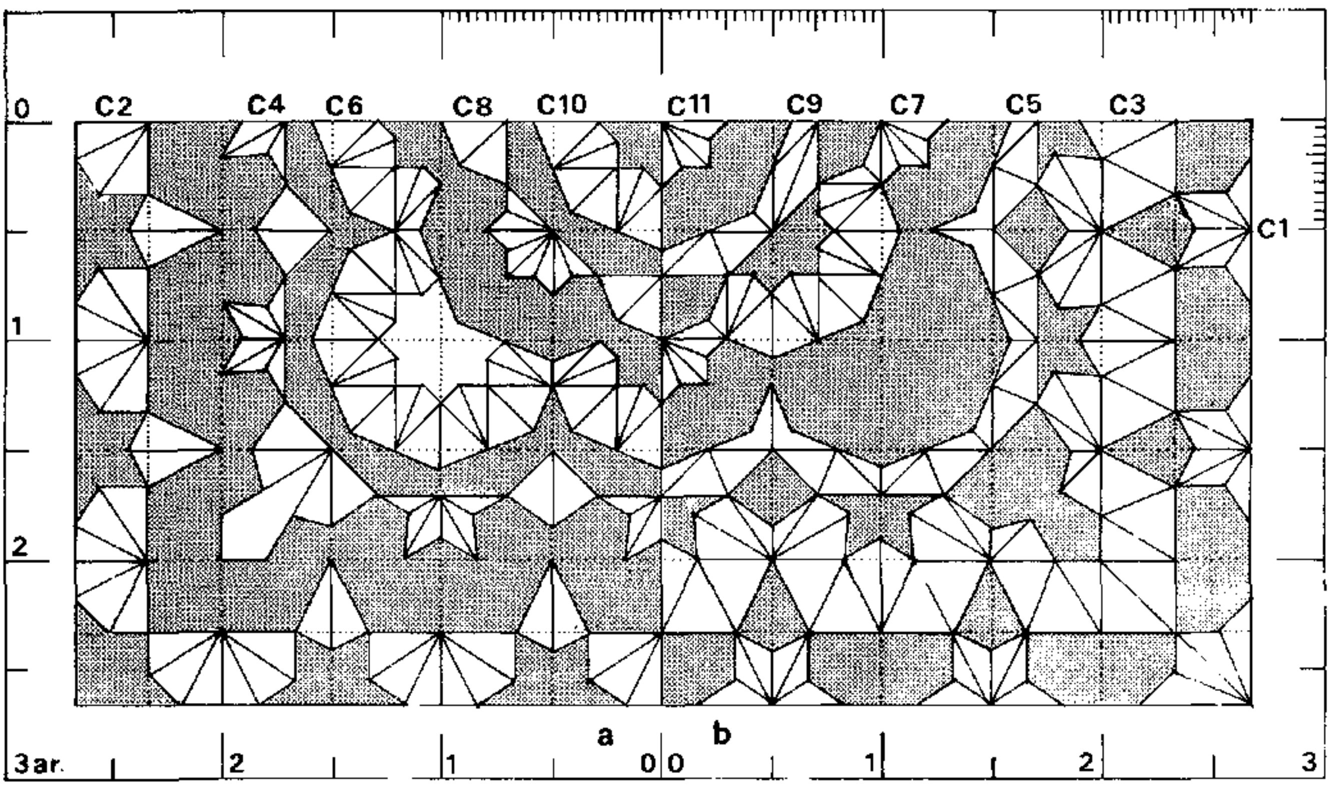

Figure 6a. Even numbered stalactite courses.

Figure 6b. Odd numbered stalactite courses.

> The plan was first made out on the floor, the outlines at the several levels being strongly marked out. Then thin slabs of stone were cut as the template of each level, and subsequently built in the wall at the required height.

These are all gypsum stalactites, but obviously a similar techique was used for the stone stalactite in Buruciye. There is no need for a separate template slab. The outline of each course could have been traced directly on stone pieces. For this purpose, the masons either used a template of paper (or a similar material) to copy on stone, or drew directly on it with the aid of approximate ratios and drawing instruments. Buzjani (Ayasofya K. 2753, 3-7) informs us that craftsmen had access to drawing instruments like *konye* (set-square), *perjal* (compass) and *mastara* (ruler). In order to fit successive stalactite courses together, each outline needs to be drawn on the top and the bottom face of vertically adjacent courses (**Figures 6a, b**).

According to our own observations in several restoration sites in Anatolia, stone pieces are carved roughly on the ground, omitting small and delicate details. Outlines of successive courses drawn on the top and bottom surfaces of each stone piece would act as guidelines for masons.

When we study the overall composition of the stalactite of Buruciye, we observe that various elements are grouped around three types of half-star formations (**Figsures 6, 7**). Each half-star corresponds to one of the three zones.

In the central zone, a half-octagram is composed of two diagonally placed squares, the side of which is equal to 10 digits (**Figure 7a**). An octagon is drawn around it utilizing the right-angled triangle 5-12-13.

It is a widely used rational triangle, known from the early periods of history. Heron of Alexandria uses it regularly for the construction of octagons (Bruins, 1964). Its tangent is equal to $(2)^{1/2}$ -1 according to the approximation series.

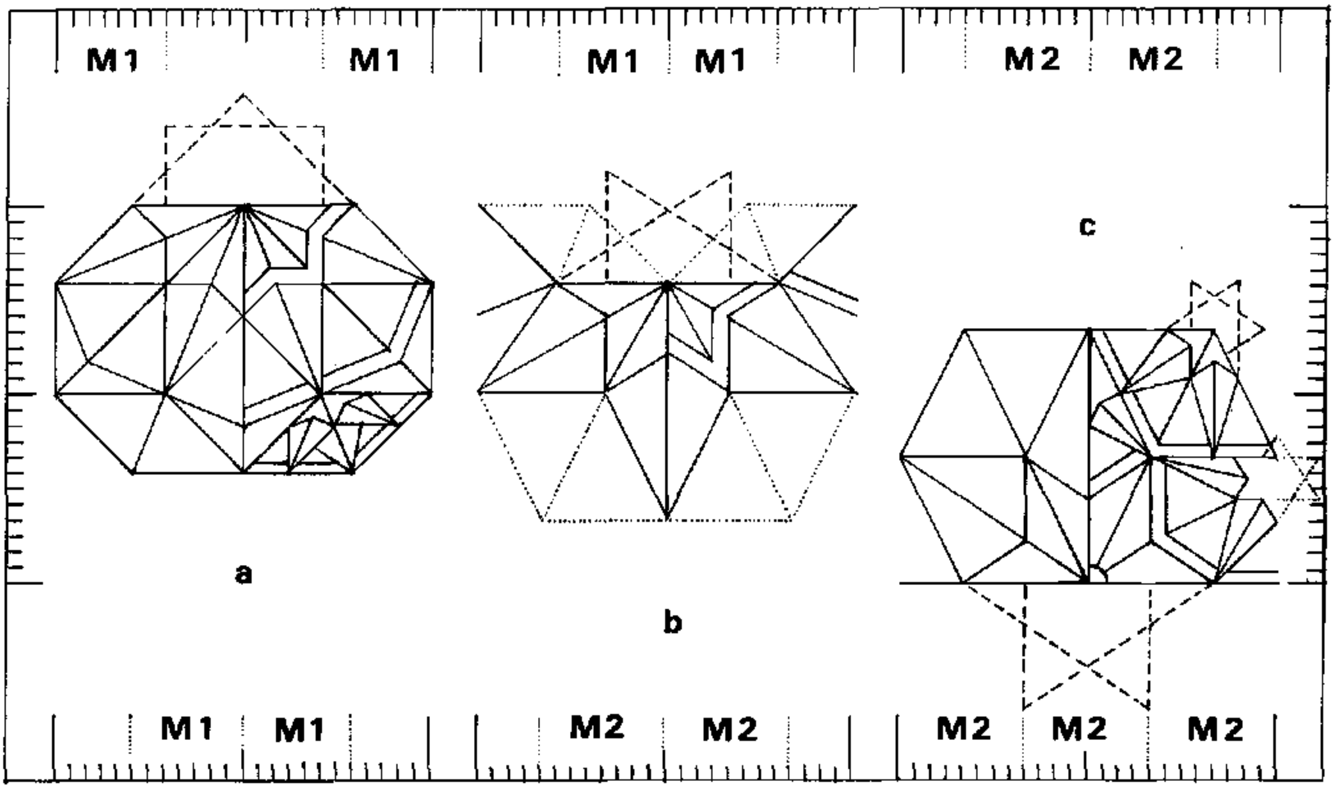

Figure 7. Star formations in three zones.

Buzjani mentions this triangle in relation to a *konye (Ayasofya K. 2753, 7)*. The distance between the centers of octagrams is equal to 17 digits which is equal to $(2)^{1/2}:2$ of the *arshin* in terms of the same approximation series, 24:17:12.

In the transition zone, a half-hexagram is formed by two equilateral triangles, the side of which is equal to 14 digits and the altitude is equal to 8 digits (**Figure7b**). 7:4 is a widely used ratio for equilateral triangles starting from Mesopotamians. In Buruciye it is used to integrate the central and the peripheral modular systems.

In the peripheral zone, a half-hexagram is formed by two isosceles triangles, the base of which is equal to 16 digits and the altitude is equal to 12 digits (**Figure 7c**). When they intersect each other along their altitude, a set of 1:2 relations are created. Similar but smaller hexagrams take place around the main one. This simple but not well known hexagram is usually mistaken for the equilateral one by modern scholars who rely on classical surveying techniques (Odekan,1977, 180).

CONSTRUCTION AND FINE DETAILS

When all the pieces of courses are roughly shaped on the ground, they are lifted up and put together as horizontal courses of the corbelling structure of the stalactite, starting from the lowest course (**Figure 8a**). Only then would masons be able to perform adjustments and elaborate details working on a scaffolding, acting like a sculptor (**Figure 8b**). This is the same process we observed in various restoration works on stalactites. We see no reason why it should be different for Buruciye.

A curious aspect of composition is the transition zone. It is left undecorated in a subtle contrast to the finely detailed peripheral and central zones (**Figure 8b**). It may be a conscious attempt to visually separate the two distinctly different geometrical zones.

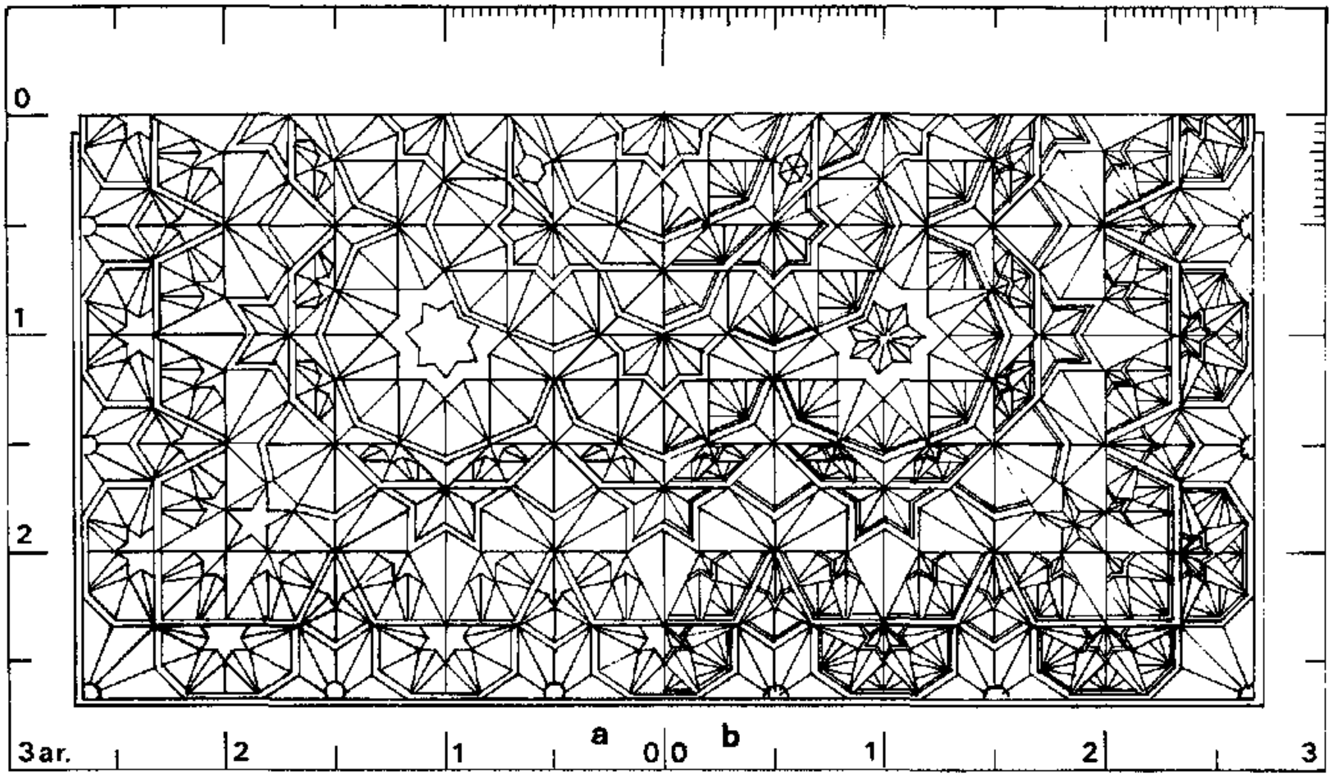

Figure 8a. Theoretical plan of the constructed stalactite (tentative).

Figure 8b. Restitution plan of the finished stalactite.

Another curious aspect of design is the pendants. Along the diagonal axes, there are two pairs of pendants fixed to the flat ceilings in the middle of star formations. The one in the central zone is a heptagram placed in a partial octagram (**Figure 8b**). Odekan (1977, C32) has mistaken it for an octagram with a missing arm; but it is a real, regular heptagram. Its diameter is equal to 8 digits and the side:radius ratio is equal to 7:8, which is the approximate ratio for $(3)^{1/2}/2$. Since a regular heptagon cannot be drawn by the aid of a compass and a straight edge, the same ratio is used by Heron (Bruins, 1964, 231). Buzjani (Ayasofya K. 2753, 35) used the same relation. The reason for using the heptagram in an alien geometric environment may, as an attractive possibility, be to indicate the use of the two modules, seven and eight digits.

The pendant in the transition zone is a special pentagram which presents an elegant solution. We believe it deserves special attention.

Usually a mystical significance is attributed to the pentagram in various cultures. It first appeared in Mesopotamia and later became the symbol of Pythagorean brotherhood. The construction of this form requires the usage of 'extreme and mean ratio' which appears in several propositions of Euclid. Plato referred to it simply as 'section' (Heath, 1956, I, 99). Today it is widely known as the 'golden section'. This irrational ratio can be expressed as an approximation series which is commonly called Fibonacci series: 5:3, 8:5, 13:8, 21:13, 34:21... The name is derived from the nickname (Filius Bonaci) of Leonardo of Pisa, who introduced it in 1202. Like the rest of his information, it is generally accepted that he borrowed this series from Muslims.

The pentagram in Buruciye can be constructed geometrically by using 'extreme and mean ratio'. But we believe the designer actually used the convenient method of Fibonacci series (7). The diagonal of a 12 digit square is equal to 17 digits (**Figure 9a**). A length of 4 digits is marked at the center of the diagonal, 2 digits on either side of the center. By this way, the diagonal is divided into three

7. The anonymous author of the Persian treatise illustrates five different constructions for the pentagon (Bulatov, 1978, 332-7). They are all approximate solutions in none of which he uses 'extreme and mean ratio', although it was well known by the mathematicians of his time, e.g. Buzjani (Ayasofya K. 2753, 34).

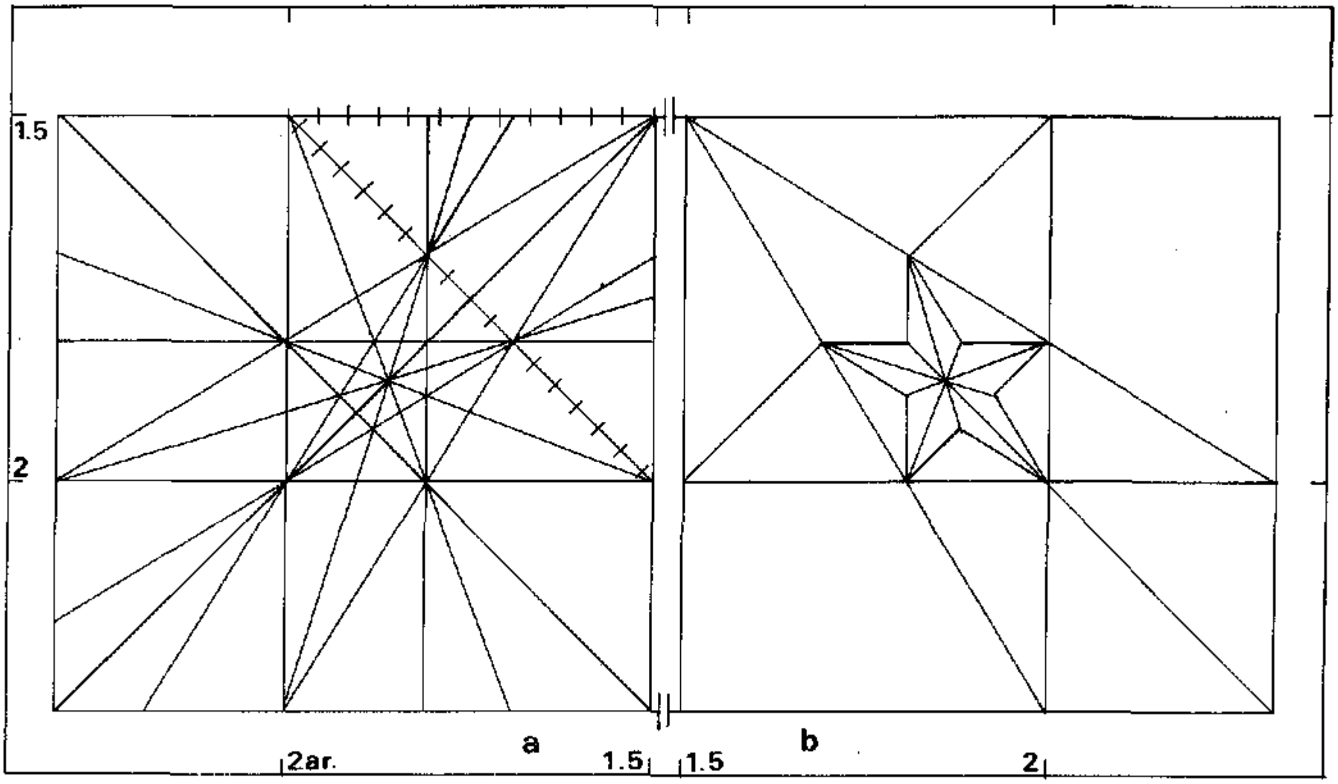

Figure 9a. Construction of the pentagram.

Figure 9b. Restitution of the pentagram pendant.

sections that are related to each other and to the whole in an hierarchical order of the Fibonacci series, 8:13:21:34. From the opposite corner of the square, two lines are drawn passing through these points, intersecting the perpendicular sides of the square and their extensions. Consequently, all the parallel and perpendicular lines are divided and related to each other according to the approximate 'extreme and mean ratio'. The result is a right-angled pentagram along the diagonal axis (**Figure 9b**).

SECTION - ELEVATION

El-Kashi explains stalactites in four categories: 'simple', 'mudded', 'arched' and 'Shirazi' (Özdural, 1990, 37-42). The elements of the stalactite of Buruciye have curved profiles and its course heights are equal to each other and twice the 'scale', M1 (**Figures 10a,b**). These are common features for both the 'arched' and the 'Shirazi' stalactites. It is difficult to fit Buruciye into a single type: uncharacteristically for an 'arched' stalactite, it has more than one geometrical system and contains pentagrams, heptagrams and pendants *shurfa*; contrary to the examples of 'Shirazi' stalactites, these pentagrams and heptagrams are not an integral part of the overall geometric system. The confusion can be explained by the fact that the stalactite development in Anatolia was independent from the one in Persia, Horasan and Transoxiana, where the 'shirazi' stalactite first appeared in the early fifteenth century (Özdural, 1990, N.22).

When the course heights of Buruciye (**Figure 10a**) are compared with the theoretical ones as defined by el-Kashi (**Figure 10b**), we observe that they correspond with each other almost precisely, except for slight deviations in certain courses (**8**). The specific profile that el-Kashi prescribes for all stalactites is not true for the ones in Buruciye.

In the lower portion, from C1 to C6, stalactite courses and stone courses correspond to each other. However, two stalactite courses are carved into single

8. El-Kashi increases the height of each course 'by a small amount' (Özdural, 1990, 42). We interpreted this 'small amount' as one digit in the case of Buruciye.

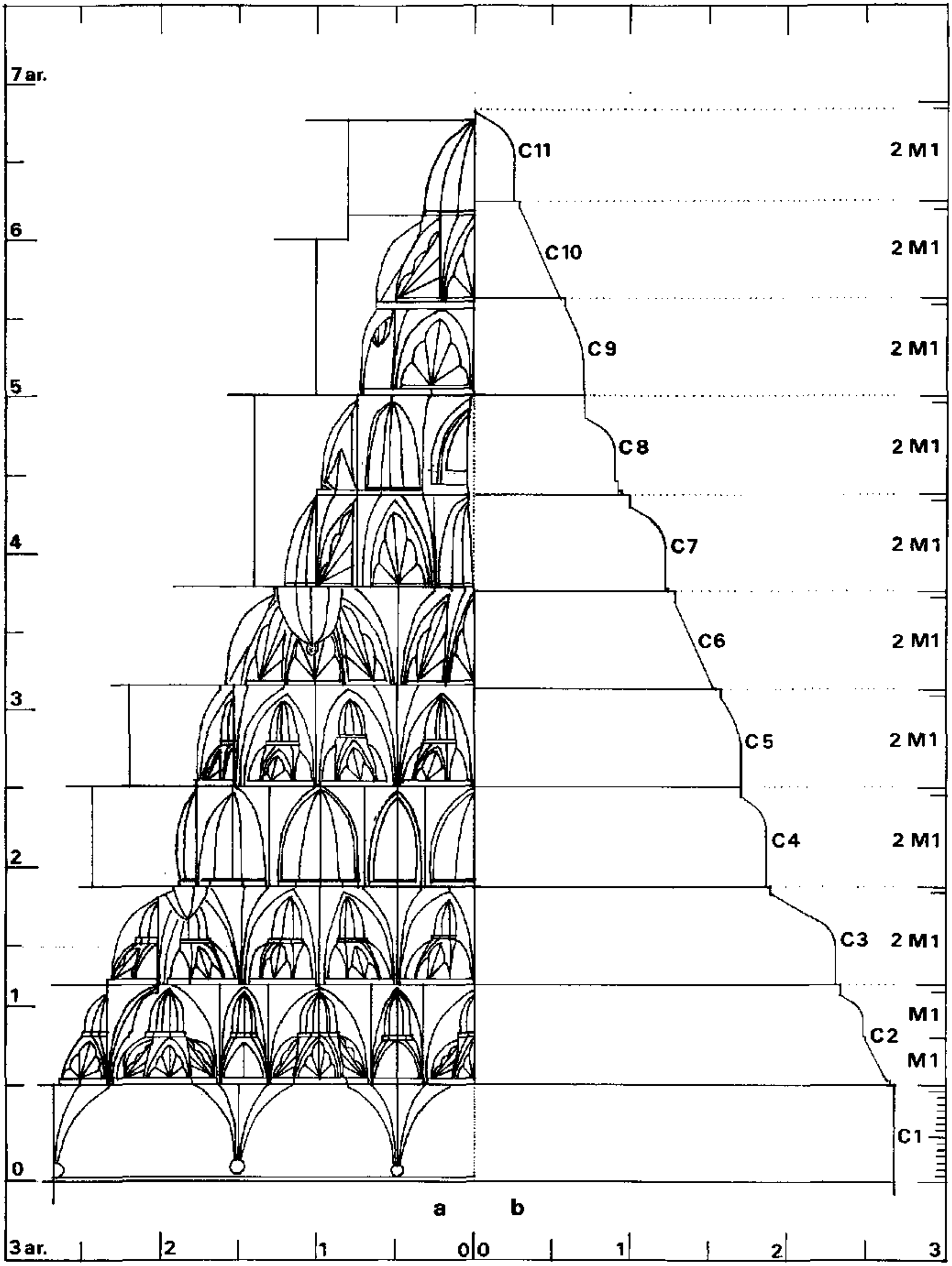

Figure 10a. Photogrammetrically surveyed elevation.

Figure 10b. Theoretical stalactite profile according to el-Kashi.

stone courses in the upper portion (C7+C8, C9+C10). When two stalactite courses are combined, it is difficult to trace and carve the horizontal course profile in the middle part. Probably as a result of this difficulty, there is a deviation from the vertical line between C7 and C8.

As a last comment we can say that, although the stalactites give very impressive spatial effects, they are not actually designed and planned three dimensionally. All the decisions are taken on the plan level; three dimensional massing is created simply by introducing course heights.

CONCLUSION

The general purpose of the present study is two-fold. Firstly, we seek to verify the validity of el-Kashi's testimony by testing it on a typical Anatolian stalactite. The result is quiet convincing. Except some minor deviations, mainly due to regional differences, the stalactite of Buruciye follows the essential rules that are observed and presented by el-Kashi. The link between el-Kashi and Buruciye can only be explained by a common tradition of stalactite design that was shared by the masons in Transoxiana and Anatolia.

Secondly, we expect to illustrate the builders' use of approximate ratios on a large scale architectural element, *e.g.* stalactite. The use of approximate ratios in medieval European architecture is convincingly argued by various scholars. In the case of the stalactite of Buruciye, too, the result is evidently, and encouragingly, worthy of credence. At this stage we can only assert that approximate ratios can be regarded as convenient means for the execution of the design of stalactites.

SİVAS BURUCİYE MEDRESESİ MUKARNASLARININ GEOMETRİK ÇÖZÜMLEMESİ

ÖZET

Alındı : 22. 2. 1993
Anahtar Sözcükler: Mimarlık Tarihi, Sivas Buruciye Medresesi, Mukarnas, Matematik Tarihi.

El-Kaşi'nin mukarnaslarla ilgili olarak bize aktardığı tipoloji, tanım ve kuralların Anadolu'daki örneklere uygunluğunu irdelemek amacıyla Sivas Buruciye Medresesi'ndeki 'portal' içi mukarnas örtüsü inceleme konusu olarak ele alınmıştır. Analiz sürecinin hazırlık aşamasını oluşturan rölöve en güvenilir teknik olan fotogrametri tekniğiyle elde edilmiştir.

Mukarnasın en alt sırasını oluşturan yarım altıgen yıldızların değişmeyen ara mesafesi olan 56 cm binada kullanılan 'arşın' olarak kabul edilmiş ve analizin ilerki aşamaları bu varsayıma dayandırılmıştır. Bu arşının 24 'parmak'a (veya 'boğun') bölündüğü düşünüldüğünde, mukarnas planının yedi parmak ve sekiz parmaklık iki modüle dayanan üç bölgeye ayrıldığı gözlenmektedir. Orta bölge 45 derecelik açı sistemine, çevresel bölge tanjantı 1/2 olan açı sistemine, ikisinin birleştiği bölge ise otuz derecelik açı sistemine dayanan bir düzenlemeyle tasarlanmıştır. Bu açı sistemlerinin gerektirdiği irrasyonel oranların, taş işçileri için çok daha pratik ve ulaşılabilir bir yol olan yaklaşık seriler aracılığıyla ifade edildiği varsayılmaktadır. Yaklaşık serilerin tarihin ilk çağlarından beri bilindiği

ve pratik alanlarda kullanıldığı yolunda elde çeşitli kaynaklar vardır. Buruciye'de bu serilerden 'kare kök 2' ilişkileri için 7/5, 'kare kök 3' ilişkileri için 7/4 oranının kullanıldığı gözlenmektedir.

Taş sıralarının önce yerde 1/1 ölçeğinde çizildiği ve hazırlandığı, bunlardan inşa edilen mukarnasın üzerinde alıştırma ve ayrıntılı süslemelerin gerçekleştirildiği düşünülmektedir. Sıra yüksekliklerinin herbirinin, el-Kaşi'nin belirttiği üzere, iki modüle eşit olduğu görülmektedir.

Buruciye mukarnas planının el-Kaşi'nin aktardığı kurallara uygunluğu ikisinin de aynı geleneğe bağlanmasıyla izah edilebilir. Öte yandan, yaklaşık oranların Buruciye'de kullanıldığının gözlenmesi bu konudaki hipotezimizin cesaret verici ilk örneği olarak kabul edilebilir.

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