DEVELOPMENT OF A TACTICAL BALLISTIC MISSILE TRAJECTORY PREDICTION TOOL

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ABSTRACT

A trajectory prediction program was developed to predict the full trajectory (partially tracked by a satellite or a radar) of a tactical ballistic missile. In order to improve the accuracy of the ballistic coefficient an adaptive C_D model is developed. Dynamic model, filtering of measurements, geocentric earth model and atmosphere model were applied and the details of the models and test results are presented. Input to the program is coordinates of the tracked trajectory portion and output is the full trajectory. In this way the launch and target point can be obtained within seconds of execution time.

Keywords: *Trajectory prediction, application of satellite measurements, ballistic model, target tracking, missile launch surveillance*

1. INTRODUCTION

It is important to detect ballistic missile launches in time and accurately for effective counter measures. Trajectory estimation plays an important role in early warning system and anti-tactical ballistic missile systems for tracking and guidance. Early detection could allow for warnings to be dispatched to the predicted impact areas and would also permit interdiction of the missile launchers. The detection of launches is also important in alerting defensive systems, such as anti-tactical ballistic missile batteries, against these attacks, as early as possible [1], [2], [3].

For these purposes missile surveillance system is needed to complete national security and to strengthen existing defense systems. Such systems may consist of satellite and/or aircraft borne radar systems.

The primary purposes of missile launch surveillance are:
1) to provide a timely report of each occurrence of a missile launch,
2) to estimate launch/trajectory parameters, and
3) to estimate present and future missile trajectories as a function of time during flight.

In the present work, the main objective is to predict launch and target points. After the missile position and velocity vectors are obtained, the prediction starts with the time integration of equations of motion using a six-parameter-state vector, which is composed of the position and

velocity vectors. Backward time integration is used for predicting the launch point on the other hand, forward time integration is used for predicting the target point. The integration procedure continues until the criterion of intersection of the missile with the Earth surface is satisfied.

2. DYNAMIC MODEL

The Ballistic Missile flies along a ballistic trajectory above the rotating Earth. Constructing a mathematical model for this system with some simplifying assumptions to describe its motion is the first step for identification and analysis. Consider a vehicle in the free-flight phase over an elliptic [4], non-rotating Earth, as illustrated in Fig. (1). In this study, the Ballistic Missile is taken as a point mass with constant weight following a ballistic trajectory. Significant forces acting on the Ballistic Missile are Drag and gravity. The drag force acts in the opposite direction to the velocity vector with a magnitude proportional to the air density (which is a function of the altitude [6]).
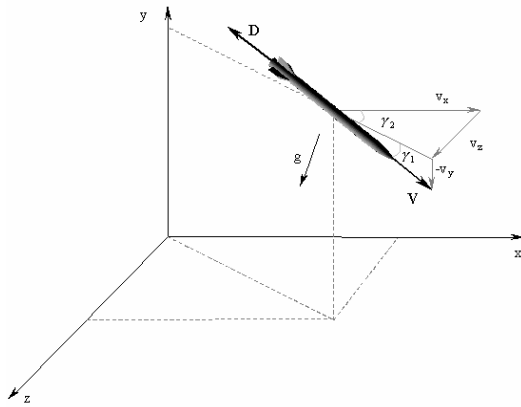
Fig. 1 Dynamic Model

The ballistic trajectory of a Ballistic Missile can be evaluated by the flight simulation programs which are using the thrust and aerodynamic data obtained from tests or analysis. While tracking a Ballistic Missile, it is considered to be difficult to estimate rocket motor size and thrust, aerodynamic and external geometry of the Ballistic Missile. The full trajectory of a tactical ballistic missile can be predicted from a tracked portion of the trajectory. In active phase of the flight, trajectory between the current location and launch point can be predicted by backward

integration in time. However target point and the trajectory directed to this point cannot be predicted properly. On the other hand, in the free-flight phase launch and target points and full trajectory can be predicted.

Trajectory Equations of Motion
The trajectory differential equations are

$$-D\cos\gamma_1\cos\gamma_2 - mg_x = ma_x \qquad (1)$$

$$-D\sin\gamma_1 - mg_y = ma_y \qquad (2)$$

$$-D\cos\gamma_1\sin\gamma_2 - mg_z = ma_z \qquad (3)$$

where D is the drag force; a_x, a_y and a_z denote acceleration as components along x, y and z respectively; g_x, g_y and g_z refer gravity components along x, y and z respectively; m is the mass of the Ballistic Missile and γ_1 - γ_2 are the flight path angles.

$$\gamma_1 = \tan^{-1}\left(-\frac{v_y}{\sqrt{v_x^2 + v_z^2}}\right) \qquad (4)$$

$$\gamma_2 = \tan^{-1}\left(\frac{v_z}{v_x}\right) \qquad (5)$$

replacing D, equation (1) becomes,

$$-\frac{1}{2}\rho V^2 \frac{SC_{D0}}{m}\cos\gamma_1\cos\gamma_2 - g_x = a_x \qquad (6)$$

In equation (6), ρ is the air density; V denotes missile velocity, S and C_{D0} represent the reference area and zero lift drag coefficient, respectively.

In equation (6), all parameters can be estimated from the ballistic trajectory data, except S, C_{D0} and m. Ballistic coefficient β is defined as,

$$\beta = \frac{SC_{D0}}{m} \qquad (7)$$

Instead of using S, C_{D0} and m, the ballistic coefficient (equation (8)) can be used to predict ballistic flight trajectory. Ballistic coefficient can then be redefined as

$$\beta = \frac{a_x + g_x}{-\frac{1}{2}\rho V^2 \cos\gamma_1\cos\gamma_2} \qquad (8)$$

using the known ballistic trajectory position data (whose existence is a must to make trajectory prediction). LSE is applied by polynomial fitting to the discrete position data in order to obtain a curve, which is a function of time. Velocity and acceleration vs. time curves can be obtained by calculating first and second derivatives respectively (equation (15)). Obviously the predicted trajectory is more accurate when it is possible to measure all trajectory parameters (position, velocity and acceleration).

$$x_t, \ t = t_0, \ldots, t_f \Rightarrow LSE \Rightarrow x(t) \Rightarrow \left. \begin{array}{c} x(t) \\ \dot{x}(t) \\ \ddot{x}(t) \end{array} \right|_{t=t_0,\ldots,t_f} \quad (15)$$

Fig. 3 Ballistic Coefficient Update Model

Ballistic Coefficient Update Model
In the literature, the same ballistic coefficient value, which is calculated for the tracked portion of trajectory, is used in the trajectory reconstruction. However, since the ballistic coefficient is a function of C_{D0}, S and m (equation (7)) and due to fact that C_{D0} changes with the flight conditions, the ballistic coefficient is no longer a constant value in the reality (Fig. (3)). Drag coefficient is the only factor that alters the ballistic coefficient during the flight.

In order to update the ballistic coefficient for each estimated trajectory location and to use an adaptive C_D model, a database that consists of C_{D0} against varying Mach number and altitude is prepared previously. For this purpose, a typical tactical ballistic missile geometry with an axisymmetric body only (without any protuberances and lifting surfaces) is used in the calculations done with Missile DATCOM [5]. Below equation, $(\beta/C_D)^S$ is a scale parameter that is obtained from the tracked trajectory portion, C_D^* denotes the current estimated drag coefficient calculated by adaptive C_D model and β^* is the updated ballistic coefficient for the current estimated trajectory location.

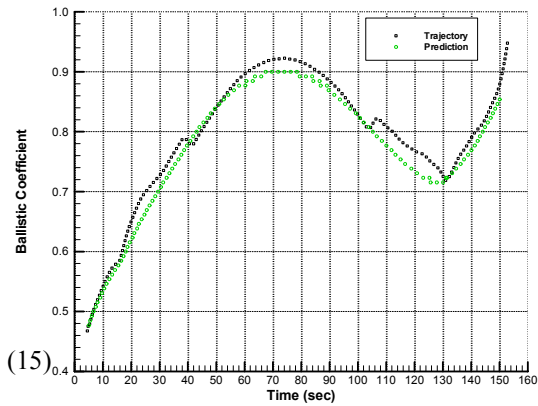$$\beta^* = \left(\frac{\beta}{C_D} \right)^S \times C_D^* \quad (16)$$

3. TEST CASES

Five different ballistic missile trajectories were reconstructed by using the flight simulation program. For each of these appropriate noises were added to the trajectory parameters in order to induce the measurement errors [7]. To simulate the real time tracking of a Ballistic Missile from a satellite or a radar, each test trajectory is split into small trajectory portions and input to the trajectory prediction program. Then the program reconstructs the trajectories corresponding to the each of the tracked trajectory portions.

Maximum ballistic trajectory parameters of these different ballistic test missiles are shown below,

Max. Range (km)	173 - 1410
Max. Altitude (km)	37 - 142
Flight Time (s)	190 - 382
Max. Velocity (m/s)	1150 - 4450

Having the longest range, Ballistic Test Missile-5 trajectory is presented. The test trajectory and a portion of it (as modeling the satellite measurement) and corresponding predicted trajectory by the program are shown in Fig. (4). The predicted trajectories around target point and launch point corresponding to each tracked portion are shown in Fig. (5) and Fig. (6) respectively. The program predicts the launch and target points within seconds of execution time. Launch and target points were predicted

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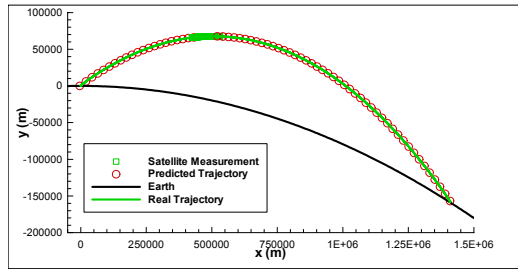accurately within the 1% error of its maximum range.
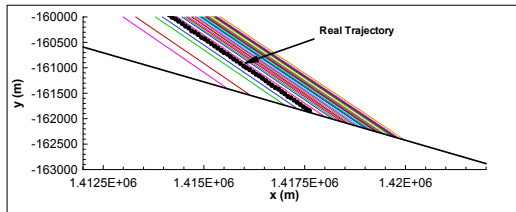

Fig. 4 Trajectory Prediction
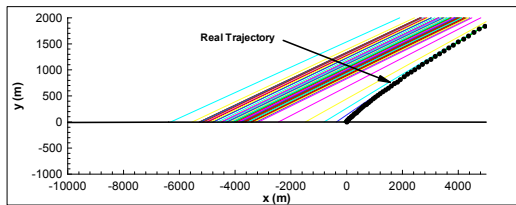

Fig. 5 Target Point Prediction

Fig. 6 Launch Point Prediction

4. CONCLUSION

In this work, a trajectory prediction program was developed to predict the full trajectory (partially tracked by satellite or radar) of a tactical ballistic missile. In order to improve the accuracy of the ballistic coefficient an adaptive C_D model is used. The prediction of launch and target point can be obtained within seconds.

Several different test cases were tested with the present program. The accuracy of predicting the launch and target points is high.

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