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EQUAL EMBEDDED ALGORITHM FOR ECONOMIC DISPATCH WITH GENERATOR CONSTRAINTS

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ABSTRACT

This paper describes Equal Embedded Algorithm (EEA) for solving Economic Dispatch (ED) problem with generator constraints. The proposed algorithm is formulated based on the conventional numerical techniques such as Interpolation and Muller method. The proposed algorithm involves selection of incremental fuel cost (lambda) values, then the output powers of generating units are obtained in terms of lambda by Newton forward interpolation and finally the evaluation of optimal lambda is done by the Muller method at required power demand from the power balance equation. The proposed algorithm has been tested on a power system having 6,15 and 40 generating units and the results of the proposed method are compared in terms of solution quality, convergence characteristics and computation time with the conventional lambda iterative method. From the case studies, it is observed that the proposed algorithm provides qualitative solution with less computational time.

Keywords: *Equal Embedded Algorithm, Economic dispatch, Newton forward interpolation, Muller method and Quadratic fuel cost function.*

1. INTRODUCTION

The main objective of Economic Dispatch (ED) problem is to determine the optimal schedule of online generating units so as to meet the power demand at minimum operating cost under various system and operating constraints. This problem is an important optimization problem in Power System Operation and is a multi model, discontinuous and highly non-linear problem due to valve point loading, ramp rate limits and prohibited operating zones. Fuel cost function of each generating unit is represented by a quadratic function in terms of output power of the generating unit.

Earlier, many classical optimization techniques such as lambda Iterative method, lambda projection method, base point and participation factors method, gradient method [8] and Dynamic Programming [1] have been applied to solve the ED problems.

In order to get the qualitative solution for solving the ED problems, Artificial Neural Network

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(ANN) techniques such as Hopfield Neural Network(HNN)[2] have been used to solve the ED problems more accurately. The objective function of the Economic Dispatch problem is transformed into a Hopfield energy function and numerical iterations are applied to minimize the energy function. The Hopfield model has been employed to solve the ED problems for units having continuous or piecewise quadratic fuel cost functions and for units having prohibited zone constraints. In the conventional Hopfield Neural Network, the input-output relationship for its neurons can be described by sigmoidal function. Due to the use of the sigmoidal function to solve the ED problems, the Hopfield model takes more iterations to provide the solution and often suffers from large computational time.

Past decade, Genetic Algorithm (GA)[3] has been used to solve the ED problems. The GA is a stochastic optimization technique, which is based on the principle of natural selection and genetics. The GA approaches are capable of solving the ill-posed problems such as non-convex functions, non-differentiable functions, local optima and multiple objectives. The GA operates on a population of candidate solutions encoded to finite bit string called chromosome. In order to get optimality, each chromosome exchanges information by using operators borrowed from natural genetic process to produce the better solution. The GA uses only the information of objective function. Recent investigations have been identified some deficiencies in the GA performance is that the cross over and mutation operations cannot ensure the better fitness of an offspring because the chromosomes in the population have similar structures and their average fitness is high toward the end of the evolutionary process.

Recently, modern heuristic methods such as Evolutionary Programming (EP) [4], Particle Swarm Optimization (PSO) [5], Ant Colony Searching Algorithm (ACSA) [6] and Tabu Search Algorithm (TSA) [7] have been used to solve the ED problems due to their ability in getting the near global optimal solution.

The main aim of the present paper is to develop a new method with reducing complexity for solving the ED problem with generator constraints by employing Equal Embedded Algorithm.

Organization of the paper is divided into five sections. Formulation of the ED problem is introduced in Section 2. The description of Equal Embedded Algorithm to solve ED problem with generator constraints is given in Section 3. Implementation of the Equal Embedded Algorithm to solve the ED problem is given in Section 4. Simulations results with various generating units are presented in Section 5. Conclusion are finally given in the last section.

2. ECONOMIC DISPATCH PROBLEM

The fuel cost function is expressed as a quadratic function of real power generation.

$$F_i(P_i) = \alpha_i + \beta_i P_i + \gamma_i P_i^2 \quad (1)$$

The objective function is

$$\text{Minimize } F_t = \sum_{i=1}^{ng} \alpha_i + \beta_i P_i + \gamma_i P_i^2 \quad (2)$$

The above objective function is subjected to various equality and inequality constraints. The equality and inequality constraints are given below.

i) Power balance equation:

$$\sum_{i=1}^{ng} P_i = P_D \quad (3)$$

(ii) Generator constraints:

$$P_i^{\min} \le P_i \le P_i^{\max} \quad (4)$$

The Lagrange function can be formulated for the ED problem from (2),(3) and (4). The expression of the lambda and the output power are

$$\lambda_i = \beta_i + 2\gamma_i P_i \quad (5)$$

$$P_i = \frac{\lambda_i - \beta_i}{2\gamma_i} \quad (6)$$

3. EQUAL EMBEDDED ALGORITHM FOR ECONOMIC DISPATCH

A new algorithm has been proposed based on numerical methods such as Interpolation and

Muller method and is known as Equal Embedded Algorithm. Formation of the Equal Embedded Algorithm is given below.

3.1. Selection of the Lambda values

Lambda values are selected from the pre-prepared power demand (PPD) table. Formation of the PPD table is given below.

1) Formation of PPD Table

The following steps are involved to form the PPD table.

Step 1 Incremental fuel costs (λ) are evaluated at minimum and maximum output power of the generating units and then the evaluated values are arranged in ascending order.

Step 2 the output powers are computed at all λ values. All λ values, output powers, sum of output powers (sop) are formulated as a table. This table is called ppd table.

Assume that N units are available. For all units, λ values are evaluated at minimum and maximum output powers of generating units. Hence, $2N$ lambda values are evaluated. The entries in the PPD table are given below.

Column 1 Arrange evaluated lambda values in ascending order.

Column 2 to N+1 Evaluated output powers of all generating units.

Column N+2 Sum of output powers at each λ.

Therefore, the dimension of the PPD table is $2N \times (N+2)$.

Last column gives the information of the range of the predicted power demands. At lowest λ value, all units are operated at minimum output powers and at highest λ all units are operated at maximum output powers. In the Economic Dispatch problem, all units must be operated in optimal manner while satisfying generating units limits.

At required power demand, the upper and lower rows of the PPD table are selected such that the power demand lies within the SOP limits and these two rows are formulated as a table and is known as Reduced PPD (RPPD) table.

3.2. Interpolation

A polynomial can be estimated from the data of known input and known output by an interpolation [10]. At desired input, the value of unknown output is evaluated from the polynomial by interpolation.

In the ED problem, the output power of generating unit is a linear relation with lambda. At this stage, lambda is taken as the input and output power of generating unit is taken as output. At required power demand, the expression of output power is obtained in terms of lambda from RPPD table by Newton forward interpolation method. The application of Newton forward interpolation method to obtain the output power of the generating unit in terms of a lambda is given below.

(i) The expression of output power in terms of lambda is given in (6).

(ii) At desired power demand λ_{j+1} , λ_j, P_i obtained from the RPPD table.

From the Table 1, output power of the generating unit is obtained in terms of lambda by using Newton forward interpolation method and the corresponding equation is

$$P_i = A_{i1} + \frac{(A_{i2} - A_{i1})}{(\lambda_{j+1} - \lambda_j)}(\lambda - \lambda_j) \quad (7)$$

Table 1. A simple RPPD table

λ	P_i	SOP= $\sum_{i=1}^{ng} P_i$
λ_j	A_{i1}	$\sum_{i=1}^{ng} A_{i1}$
λ_{j+}	A_{i2}	$\sum_{i=1}^{ng} A2$

From the interpolation method, all output powers of generating units are obtained in terms of the lambda.

3.3. Muller method

In Muller method, higher order polynomial is approximated by a second-degree curve in the surrounding of a root. The roots of the quadratic equation are then assumed approximately the roots of the equation. This method is iterative and converges almost quadratically[11].

Let x_{i-2}, x_{i-1}, x_i are three distinct approximations to a root of $f(x)=0$ and y_{i-2}, y_{i-1} *and* y_i are the corresponding values of $y = f(x)$. The relation between y and x can be represented by

$$y = A(x - x_i)^2 + B(x - x_i) + y_i \quad (8)$$

Where

$$A = \frac{(x_{i-2} - x_{i-1})(y_{i-1} - y_i) - (x_{i-1} - x_i)(y_{i-2} - y_i)}{(x_{i-1} - x_{i-2})(x_{i-1} - x_i)(x_{i-2} - x_i)}$$

$$B = \frac{(x_{i-2} - x_i)^2(y_{i-1} - y_i) - (x_{i-1} - x_i)^2(y_{i-2} - y_i)}{(x_{i-1} - x_{i-2})(x_{i-1} - x_i)(x_{i-2} - x_i)}$$

$$x_{i-1}^{(1)} = x_{i-1}^{(0)} - \frac{2\,.y_i}{B \pm \sqrt{B^2 - 4Ay_i}} \quad (9)$$

The sign in the denominator should be chosen properly so as to get the denominator is largest in magnitude. With this choice, equation (9) gives the next approximation to the root.

The application of Muller method to find the lambda value from the power balance equation at required power demand in the ED problem is as follows:

$$\text{f}(\lambda) = \sum_{i=1}^{ng} P_i(\lambda) - P_D \quad (10)$$

The above equation is highly nonlinear equation and the solution can be get by the Muller method. At the required power demand,

$$x_{i-2} = \lambda_j \ \&\text{y}_{\text{i-2}} = SOP_j \quad (11)$$

$$x_i = \lambda_{j+1} \ \&\text{y}_{\text{i}} = SOP_{j+1} \quad (12)$$

$$x_{i-1} = (\lambda_j + \lambda_{j+1})/2 \quad (13)$$

From (9), the lambda value can be evaluated by an iterative approach. The chief advantage of the Muller method is that it converges quadratically to find the root of the polynomial.

4. IMPLEMENTATION OF EQUAL EMBEDDED ALGORITHM TO SOLVE THE ECONOMIC DISPATCH PROBLEM

Implementation of the Equal Embedded Algorithm to solve the ED problem is given below.

Step 1 PPD table is formulated.
Step 2 At required power demand, the RPPD table is obtained.
Step3 The expressions of output powers of the generating units are obtained in terms of the lambda by Newton forward interpolation from RPPD table.
Step 4 Evaluation of optimal lambda is done by the Muller method from the power balance equation
Step 5 The optimal solution is obtained.

5. CASE STUDIES AND SIMULATION RESULTS

The proposed algorithm has been implemented in MATLAB (Version 7.0) and executed on Pentium IV, 2.4 GHz personal computer with 256 MB RAM to solve the ED problem of a power system having 6,15 and 40 generating units with the generator constraints.

The simulation results obtained from the proposed method are compared in terms of the solution quality, convergence characteristics and computation time with the lambda iterative method.

During the execution of lambda iterative method, the initial lambda value is the lowest lambda value among all lambda values of the generating units at their minimum and maximum output powers.

5.1. Case studies

1) **6- units system**

In this case study, the system contains six thermal units that are given in [9]. The fuel cost data of the six thermal units is given in Table 2.

Table 2. Fuel cost data of six units system.

U	α_i (\$)	β_i (\$/MW)	γ_i (\$/MW^2)	P_i^{min} (MW)	P_i^{max} (MW)
1	240	7	0.007	100	500
2	200	10	0.0095	50	200
3	220	8.5	0.009	80	300
4	200	11	0.009	50	150
5	220	10.5	0.008	50	200
6	190	12	0.0075	50	120

In this case the initial lambda is taken as 8.4 for the lambda iterative method.

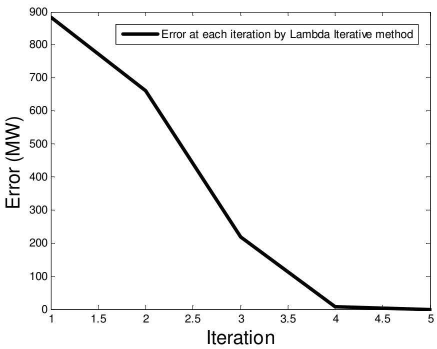

Figure 1. Convergence characteristic of the lambda iterative method of 6 units system at the power demand of 1263 MW

The convergence characteristic of the lambda iterative method of 6 units system is shown in Fig 1.

It is well known that the optimal solution is depends on the initial lambda. Fig 1 shows that the lambda iterative algorithm takes 5 iterations to get the optimal solution. But, the proposed method provides the optimal solution without any iteration.

In this case, the results of the proposed method are compared with the lambda iterative method and the simulation results are given in Table 3.

Table 3. Optimum solution of each unit at 1263 MW by lambda iterative method and proposed method

	METHODS	
	LAMBDA ITEATIVE	PROPOSED
Optimal Lambda (\$/MW)	13.253901	13.253901
P_1 (MW)	446.7073	446.7073
P_2 (MW)	171.258	171.258
P_3 (MW)	264.105	264.105
P_4 (MW)	125.216	125.216
P_5 (MW)	172.118	172.118
P_6 (MW)	083.593	083.593
Fuel cost (\$)	15275.93	15275.93
No of Iterations	5	2
Computational time (Sec)	0.031	0.001

2) **15- units system**

In this case, the power system consists of 15 thermal units and the data is extracted from [5] and given in Table 4.

Table 4. Fuel cost data of 15 units system

U	α_i (\$)	β_i (\$/MW)	γ_i (\$/MW^2)	P_i^{min} (MW)	P_i^{max} (MW)
1	671	10.1	0.000299	150	455
2	574	10.2	0.000183	150	455
3	374	8.8	0.001126	20	130
4	374	8.8	0.001126	20	130
5	461	10.4	0.000205	150	470
6	630	10.1	0.000301	135	460
7	548	9.8	0.000364	135	465
8	227	11.2	0.000338	60	300
9	173	11.2	0.000807	25	162
10	175	10.7	0.001203	25	160
11	186	10.2	0.003586	20	80
12	230	9.9	0.005513	20	80
13	225	13.1	0.000371	25	85
14	309	12.1	0.001929	15	55
15	323	12.4	0.004447	15	55

In this case also the results of proposed method are compared with the lambda iterative method.

The convergence characteristic of the lambda iterative method is shown in Fig 2.

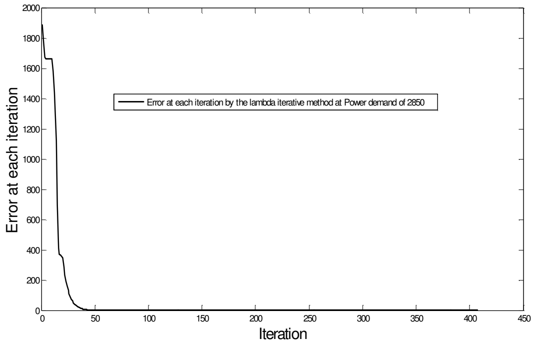

Figure 2. Convergence characteristic of lambda iterative method at the Power demand of 2850 MW.

Fuel costs, iterations and Time at different power demands by Lambda iterative method and proposed method are shown in Table 5. It is clear from the Table 5 that the proposed method provides optimal solution in less computational time compared to lambda iterative method. Number of iterations is changing with respect to change in the power demand. But, the proposed method provides the optimal solution in two iterations irrespective of the power demand.

Table 5. Fuel costs, iterations and Time at different power demands by Lambda iterative method and proposed method

	Power demand (MW)	Method	Fuel cost ($)	Itera tions	Time (Sec)
1	1850	LIM	24182.58	51	0.063
		Proposed	24182.58	2	0.031
2	2250	LIM	28303.47	43	0.063
		Proposed	28303.47	2	0.031
3	2450	LIM	30373.65	49	0.063
		Proposed	30373.65	2	0.031
4	2630	LIM	32256.66	79	0.063
		Proposed	32256.66	2	0.031
5	2850	LIM	34578.18	407	0.078
		Proposed	34578.18	2	0.031
6	3020	LIM	36424.8	382	0.096
		Proposed	36424.8	2	0.031

3) **40 units system**

In this case, the proposed algorithm and lambda iterative algorithm are applied for 40 generating units [3] system. The fuel cost data of the 40 units system is given in Table 6.

Table 6. Fuel cost data of 40 units system

U	α_i ($)	β_i ($/MW)	γ_i ($/MW^2)	P_i^{min} (MW)	P_i^{max} (MW)
1	170.44	8.336	0.03073	40	80
2	309.54	7.0706	0.02028	60	120
3	369.03	8.1817	0.00942	80	190
4	135.48	6.9467	0.08482	24	42
5	135.19	6.5595	0.09693	26	42
6	222.33	8.0543	0.01142	68	140
7	287.71	8.0323	0.00357	110	300
8	391.98	6.999	0.00492	135	300
9	455.76	6.602	0.00573	135	300
0	722.82	12.908	0.00605	130	300
11	635.2	12.986	0.00515	94	375
12	654.69	12.796	0.00569	94	375
13	913.4	12.501	0.00421	125	500
14	1760.4	8.8412	0.00752	125	500
15	1728.3	9.1575	0.00708	125	500
16	1728.3	9.1575	0.00708	125	500
17	1728.3	9.1575	0.00708	125	500
18	647.85	7.9691	0.00313	220	500
19	649.69	7.955	0.00313	220	500
20	647.83	7.9691	0.00313	242	550
21	647.81	7.9691	0.00313	242	550
22	785.96	6.6313	0.00298	254	550
23	785.96	6.6313	0.00298	254	550
24	794.53	6.6611	0.00284	254	550
25	794.53	6.6611	0.00284	254	550
26	801.32	7.1032	0.00277	254	550
27	801.32	7.1032	0.00277	254	550
28	1055.1	3.3353	0.52124	10	150
29	1055.1	3.3353	0.52124	10	150
30	1055.1	3.3353	0.52124	10	150
31	1207.8	13.052	0.25098	20	70
32	810.79	21.887	0.16766	20	70
33	1247.7	10.244	0.2635	20	70
34	1219.2	8.3707	0.30575	20	70
35	641.43	26.258	0.18362	18	60
36	1112.8	9.6956	0.32563	18	60
37	1044.4	7.1633	0.33722	20	60
38	832.24	16.339	0.23915	25	60
39	834.24	16.339	0.23915	25	60
40	1035.2	16.339	0.23915	25	60

In order to prove the applicability of the proposed method, Large-scale systems such as 80 and 120 generating units have been considered. The fuel cost data of 80 and 120 units are two times and three time of fuel cost data of 40 units system.

The simulation results in terms of fuel cost and iterations of the Lambda iterative method and the proposed method are given in Table 7.

Table 7. Simulation results of lambda iterative method and proposed method for 40,80 and 120 unit system

S.no	No of Units	Power demand	Method	Fuel cost ($)	Itera tions
1	40	10500	LIM	143934.67	66
			Proposed	143934.67	2
2	80	21000	LIM	287869.347	69
			Proposed	287869.347	2
3	120	31500	LIM	431804.02	49
			Proposed	431804.02	2

5.2. Comparıson Of Methods

1. Solution quality

Tables-3,5,7 demonstrate the effectiveness of the proposed method for getting the qualitative solution compared to lambda iterative method. The proposed method yields better solution with considerable computational time.

In lambda iterative method, the quality of solution based on error that is the difference between the generated power and power demand. Generally, the fixation of the tolerance is absolute value of 0.01 for fast convergence. But, the optimal solution of lambda in the proposed method is root of the power balance equation. It indicates that the proposed method provides qualitative solution with accurate.

2. Convergence Characteristics

The proposed method provides the qualitative solution within few iterations. At any power demand, the proposed method converges within 2 iterations. But, the lambda iterative method takes more iterations to give the qualitative solution. Also, one more difficulty in the lambda iterative method is that the suitable value of initial lambda is required Usually, number of iterations is increases with the increase of system size in lambda iterative method.

3. Computational time

Due to less iterations, the proposed method has better computation performance than the lambda iterative method. The evaluation process involved in the proposed method is that the power expressions in terms of lambda by interpolation and the evaluation of the lambda is done by Muller method from the power balance equation. The computational time of the proposed method is very less because the concepts used in the proposed method are conventional methods.

6. CONCLUSION

Equal Embedded Algorithm has been proposed in this paper for solving the Economic Dispatch problem of a power system having 6,15 and 40 generating units with generator constraints. Salient feature of the proposed method is that it gives high quality solution with fast convergence characteristics compared to the lambda iterative method. Irrespective of the system size, the proposed method converges within 2 iterations at any power demand. This aspect is useful for solving large-scale Economic Dispatch problems. The simulation results show that the proposed method is capable to solve the Economic Dispatch problems for large-scale systems.

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