ANALYSIS OF SELECTION DIVERSITY ON THE MATCHED FILTER BOUND OF BPSK ON MULTIPATH RAYLEIGH FADING CHANNELS

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Abstract: The impact of diversity combining on the matched filter bound of BPSK on time-discrete multipath slowly Rayleigh fading intersymbol interference channels is analysed. Expressions for the bound are derived for *SNR* selection, *S+N* selection, and also equal gain diversity combining. Numerical results for the derived bounds are presented on a GSM typical urban channel model.

Keywords: Rayleigh channels, Diversity, Matched filters, BPSK.

Çok Yollu Rayleigh Sönümlü Kanallarda Seçimlik Çeşitlemenin İkili Evre Kaydırmalı Anahtarlamaya İlişkin Uygunlaştırılmış Süzgeç Sınırı Üzerindeki Etkisinin Analizi

Özet: Ayrık zamanlı çok yollu yavaş Rayleigh sönümlü sembollerarası girişimli kanallarda çeşitleme birleştirmenin ikili evre kaydırmalı anahtarlamaya ilişkin uygunlaştırılmış süzgeç sınırı üzerindeki etkisi analiz edilmiş, SNR seçimli, S+N seçimli ve eşit kazanç tabanlı çeşitleme birleştirme durumlarına ilişkin sınır ifadeleri türetilmiştir. Elde edilen ifadelere ilişkin sayısal sonuçlar, tipik bir şehir içi GSM kanal modeli kullanılarak sunulmuştur.

Anahtar Kelimeler: Rayleigh kanallar, Çeşitleme, Uygunlaştırılmış süzgeçler, BPSK.

1. INTRODUCTION

Matched filter bound (MFB) is a useful tool for assessing the performance of a receiver operating in fading channels. It is determined by assuming the transmission of a single interference-free pulse and averaging the bit error rate (BER) of perfect matched filter over the fading statistic. Diversity is an effective technique to mitigate the destructive effects of channel fading, and the selection diversity is the most practical one amongst the others (Paulraj, 1999). Previous studies have evaluated the MFB on slowly fading intersymbol interference (ISI) channels in Mazo (1991), Clark, Greenstein, Kennedy, and Shafi (1992), Kaasila and Mämmelä (1994), Ling (1995), Nicholas and Taylor (2001) for binary phase shift keying (BPSK), and also for different modulation format and transmission conditions in Burchill and Leung (1995), Kim, Kim, Jeong, and Lee (1997). However, only Clark, Greenstein, Kennedy, and Shafi (1992) and Ling (1995) considered the impact of diversity combining on the MFB. But their analysis are limited to only the equal gain combining case, for which only Clark, Greenstein, Kennedy, and Shafi (1992) provides a direct expression.

In this paper, we extend the diversity analysis on the MFB of BPSK to *SNR* (signal-to-noise ratio) selection and the recently proposed *S+N* (signal-plus-noise) selection diversity (Neasmith and Beaulieu, 1998) on time-discrete multipath Rayleigh fading ISI channels by adopting the model in Ling (1995), and present a bound example for a typical urban GSM channel model. For convenience and comparison with consistence of notation, we also present a bound expression for the equal gain combining (EGC) case as an alternative to that given in Clark, Greenstein, Kennedy, and Shafi (1992).

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2. SYSTEM MODEL

We consider the time-discrete L-path complex baseband multipath Rayleigh fading channel in Ling (1995) with statistically independent M branch diversity reception with unequal average SNR, as equal powers in diversity branches are rarely available (Paulraj, 1999, Annamalai, 1999). It is supposed that the diversity branches have the same multipath delays and shape of multipath intensity profile. Assuming slow-fading with a perfect knowledge of the channel and coherent demodulation, noise-free matched filter output on the mth branch for a transmitted single bit, 1, Y_m, which represents twice the received bit energy, may be written as (Ling, 1995)

$$Y_m = \sum_{i=1}^{L}\sum_{j=1}^{L} \alpha_{m,i}\alpha_{m,j} z_{m,i} z_{m,j}^{*} R(\tau_i - \tau_j) = \sum_{i=1}^{L} \lambda_{m,i} \left|\psi_{m,i}\right|^2 \tag{1}$$

where $\alpha_{m,i}$ is the root-mean-square value of the magnitude of the ith path, $z_{m,i}$ is an independent zero-mean unit variance complex Gaussian random variable, $R(t)$ is the autocorrelation function of the transmitter shaping pulse, τ_i is the multipath delay, $\lambda_{m,i}$'s are the eigenvalues of LxL non-negative definite Hermitian matrix **M** with elements $\{m_{ij}\} = \alpha_{m,i}\alpha_{m,j} R(\tau_i - \tau_j)$, and $\psi_{m,i}$'s are zero-mean unit variance complex Gaussian random variables on the mth branch. Y_m is then a χ^2 distributed random variable, whose probability density function (pdf) is, assuming distinct eigenvalues, given by

$$f_{Y_m}(y_m) = \sum_{i=1}^{L} \frac{p_i}{\lambda_{m,i}} e^{-y_m/\lambda_{m,i}} \quad , \quad y_m \geq 0 \tag{2}$$

where $p_i = \prod_{k=1,\ k\neq i}^{L} \lambda_{m,i} / (\lambda_{m,i} - \lambda_{m,k})$. The noise sample at the matched filter output, N_m, is a zero mean Gaussian random variable with variance $\sigma_{N_m}^2 = N_0 y_m$ and independent of the fading process, where N_0 is the power spectral density of the channel noise. The composite signal sample at the matched filter output is therefore $Z_m = Y_m + N_m$. We note that the average SNR for mth branch is $\bar{\gamma}_m = E\{Y_m\}/2N_0 = \frac{1}{2N_0}\sum_{j=1}^{L}\lambda_{m,j}$, and the normalised eigenvalues for the branches are the same, $\bar{\lambda}_{m,i} = \lambda_{m,i} / \sum_{j=1}^{L}\lambda_{m,j} = \bar{\lambda}_i$, where $m = 1,\ldots,M$, $i = 1,\ldots,L$. Transmitted bits are assumed equiprobable.

3. SNR SELECTION DIVERSITY

In this classical selection diversity scheme, the branch providing the largest SNR is chosen among the M diversity branches. Let the maximum of the output instantaneous SNR be $\Gamma = \max\{\gamma_1, \gamma_2, \ldots\ldots, \gamma_M\}$ where $\gamma_i = Y_i / 2N_0$. Following Paulraj (1999), Stüber (1999), the pdf of Γ can be expressed as

$$f_\Gamma(\gamma) = \sum_{m=1}^{M}\sum_{i=1}^{L} \frac{p_i}{\bar{\lambda}_i \bar{\gamma}_m} \exp\left(\frac{-\gamma}{\bar{\lambda}_i \bar{\gamma}_m}\right) \times \prod_{\substack{k=1 \\ k\neq m}}^{M} \sum_{j=1}^{L} p_j \left(1 - \exp\left(\frac{-\gamma}{\bar{\lambda}_j \bar{\gamma}_k}\right)\right) \tag{3}$$

The P_b can then be obtained by a numerical evaluation of $P_b = \frac{1}{2}\int_0^\infty erfc\left(\sqrt{\gamma}\right) f_\Gamma(\gamma)\, d\gamma$.

4. S+N SELECTION DIVERSITY

As proposed in Annamalai (1999), the branch with the maximum composite signal level (signal plus noise) is chosen among the M branches. The P_b in this case may be formulated as $P_b = 1-\sum_{m=1}^{M}\Pr\left\{z_m > 0,\, |z_m| > |z_i|,\, i \neq m,\, i = 1,\ldots,M\right\}$. That is,

$$P_b = 1-\sum_{m=1}^{M}\int_0^{\infty} f_{Z_m}(z_m)\prod_{\substack{i=1\\ i\neq m}}^{M} F_{|Z_i|}(z_m)\, dz_m \tag{4}$$

where $F_{|Z_i|}(z_m) = \int_{-z_m}^{z_m} f_{Z_i}(z_i)\, dz_i$. Now, we need to find the pdf of Z_m, the matched filter output for the mth branch. By using $f_{Z_m}(z_m) = \int_0^{\infty} f_{Z_m}(z_m \mid y_m) f_{Y_m}(y_m)\, dy_m$ with (2), where

$f_{Z_m}(z_m \mid y_m) = \dfrac{1}{\sqrt{2\pi N_0 y_m}}\exp\left(-(z_m - y_m)^2 / 2N_0 y_m\right)$, we obtain the pdf of Z_m as

$$f_{Z_m}(z_m) = \sum_{i=1}^{L}\frac{p_i}{\lambda_{m,i}}\sqrt{\frac{\bar{\lambda}_i\bar{\gamma}_m}{1+\bar{\lambda}_i\bar{\gamma}_m}}\exp\left[\left(z_m - |z_m|\sqrt{\frac{1+\bar{\lambda}_i\bar{\gamma}_m}{\bar{\lambda}_i\bar{\gamma}_m}}\right)/N_0\right] \tag{5}$$

Therefore, using (5) in (4), we find

$$P_b = 1-\sum_{m=1}^{M}\int_0^{\infty}\sum_{k=1}^{L}\frac{p_k}{\lambda_{m,k}K_1}e^{-z_m(K_1-1)/N_0}\prod_{\substack{i=1\\ i\neq m}}^{M}\sum_{j=1}^{L}\frac{p_j}{2\bar{\lambda}_j\bar{\gamma}_i K_2}\left[\frac{1-e^{-z_m(K_2+1)/N_0}}{K_2+1}+\frac{1-e^{-z_m(K_2-1)/N_0}}{K_2-1}\right] dz_m \tag{6}$$

where $K_1 = \sqrt{1+1/\left(\bar{\lambda}_k\bar{\gamma}_m\right)}$ and $K_2 = \sqrt{1+1/\left(\bar{\lambda}_j\bar{\gamma}_i\right)}$.

5. EQUAL GAIN COMBINING

EGC here is equivalent to the maximal ratio combining (MRC), and thus the optimum way of combining the matched filter outputs. With M branch combining, the received signal sample is $Y = \sum_{m=1}^{M} Y_m$ with the pdf $f_Y(y) = \sum_{m=1}^{M}\sum_{i=1}^{L}\frac{p_{m,i}}{\lambda_{m,i}}e^{-y/\lambda_{m,i}},\; y \geq 0$, where $p_{m,i} = \prod_{k=1}^{M}\prod_{j=1}^{L} 1/(1-\lambda_{k,j}/\lambda_{m,i})$ in which $j \neq i$ for $k = m$, the noise sample $N = \sum_{m=1}^{M} N_m$ is a zero mean Gaussian random variable with variance $\sigma_N^2 = N_0 y$, and the decision statistic is $Z = Y + N$. Without resorting to error function, we directly obtain the pdf of Z, $f_Z(z)$, by following the way of finding (5), and express the MFB on the BER as

$$P_b = \int_{-\infty}^{0} f_Z(z)\, dz = \frac{1}{2}\sum_{m=1}^{M}\sum_{i=1}^{L}\frac{p_{m,i}}{1+\bar{\lambda}_i\bar{\gamma}_m+\sqrt{\bar{\lambda}_i\bar{\gamma}_m+\left(\bar{\lambda}_i\bar{\gamma}_m\right)^2}} \tag{7}$$

The result of the expression in (7) agrees with that given in Clark, Greenstein, Kennedy, and Shafi (1992).

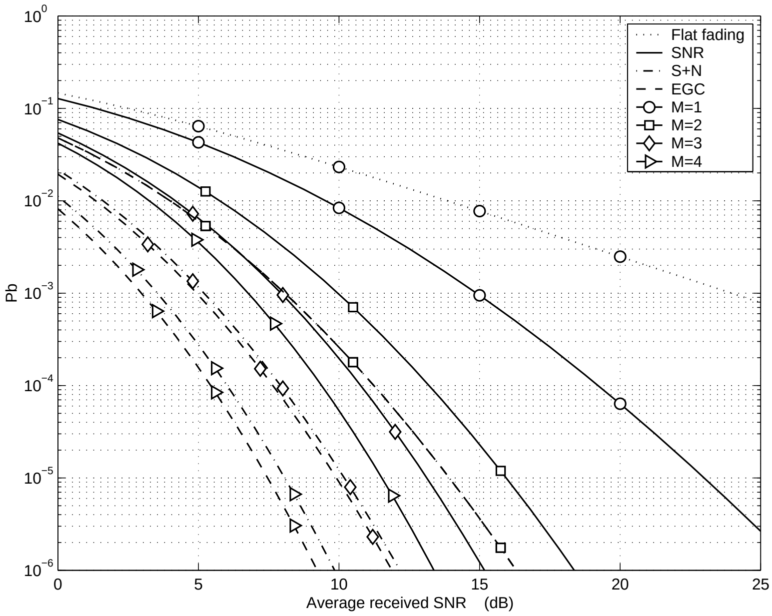

Figure 1.
MFB of BPSK with diversity combining.

6. RESULTS AND DISCUSSION

We use the GSM typical urban channel model described in Ling (1995), and present numerical results for the MFBs derived in this paper for various orders of diversity combining in Figure 1, in which only the identical average received *SNR* case for the branches is included. It is assumed that the transmitter filter has square-root raised cosine frequency response with roll-off factor 0.35. Normalised nontrivial eigenvalues for the channel at the GSM symbol rate are 0.8671, 0.1204, 0.0119, and 0.0005 (Ling, 1995). The expressions given in the paper are derived for the distinct eigenvalue case due to the shortage of space, however, the results can be extended to include eigenvalue multiplicity. MFB for the flat Rayleigh fading with no diversity is also shown in the figure. Compared with the flat fading, the implicit multipath delay diversity advantage with *M=1* is clearly seen from the figure (the uppermost two curves). It is observed that the performance of *S+N* selection is superior to conventional *SNR* selection diversity, which agrees with Neasmith and Beaulieu (1998), but is outperformed by the EGC (or equivalently MRC) scheme. Nevertheless, EGC and the *S+N* selection perform identically for dual diversity, which is intuitively satisfying since the sign of the sum in EGC (and hence the P_b) is determined by the largest composite signal level employed by the *S+N* selection scheme. Furthermore, their performances are comparable for *M=3*. However, the difference in performance increases with the increasing order of diversity.

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