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DNR TABANLI ÖĞRETİME GÖRE MATEMATİK ÖĞRETMEN ADAYLARININ İSPAT ŞEMALARININ İNCELENMESİ

INVESTIGATION OF PRESERVICE MATHEMATICS TEACHERS' PROOF SCHEMES ACCORDİNG TO DNR BASED INSTRUCTION

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Abstract

The aim of this study is to determine the types of the proof schemes used in the proof process by freshman and senior elementary pre-service mathematics teachers who are students at public university in 2011-2012 academic year and also to examine whether there is any difference between proof schemes used by freshman and senior elementary pre-service mathematics teachers. The sample of the study consists of 135 elementary pre-service mathematics teachers. Because the actual state is investigated descriptive method is used. 5 problems are asked and answers are classified according to proof schemes. Classification designed by Harel and Sowder (1998) is used. In the analysis process, answers of elementary pre-service mathematics teachers are evaluated according to proof schemes' properties and classified into 4 categories; external, empirical, analytical, and empty. Percentage frequency tables of proof schemes prepared and bar chart is drawn.

Also, it is analyzed that whether there is a significant difference between proof schemes according to class levels or not by using t test with a statistical programme. Results indicated that all proof scheme types (external, analytical, empirical) are used by the students. Also, empirical schemes are mostly used by freshman elementary pre-service mathematics teachers and analytical schemes are mostly used by senior elementary pre-service mathematics teachers. Suggestions made according to results of the study.

***Key Words*:** Proof, Proof Schemes, Pre-service Mathematics Teachers

Öz

Bu çalışmanın amacı 2011-2012 öğretim yılında bir devlet üniversitesinin ilköğretim matematik öğretmenliği programında öğrenim gören birinci ve son sınıf öğretmen adaylarının problem çözerken kullandıkları ispat şemalarını belirlemek ve ayrıca birinci ve son sınıf matematik öğretmen adaylarının kulladığı ispat şemaları arasında anlamlı bir farklılık olup olmadığını araştırmaktır. Çalışma toplam 135 öğretmen adayı ile yürütülmüştür. Mevcut durum betimlendiğinden dolayı tarama modeli kullanılmıştır. Öğrencilere genel matematik dersine yönelik, sınıf seviyelerine uygun 5 problem sorulmuş ve bu sorulara verdikleri yanıtların dışsal, deneysel ve analitk ispat şemalarından hangisine ait olduğu belirlenmiştir. İspat şemalarının belirlenmesinde Harel ve Sowder (1998) tarafından oluşturulan sınıflama kullanılmıştır. Verilerin analiz sürecinde, ispat şemalarının özellikleri göz önünde bulundurularak ilköğretim matematik öğretmen adaylarının problemlere ilişkin çözümleri değerlendirilmiş ve dışsal, deneysel, analitik ve boş olmak üzere dört kategori başlığı altında toplamıştır. İspat şemalarına ilişkin yüzde frekans tablosu oluşturularak sütun grafiği çizilmiştir. Sınıf seviyesine göre öğretmen adaylarının kullandığı ispat şemalarında anlamlı bir farklılık olup olmadığı t testi yapılarak incelenmiştir. Elde edilen sonuçlar öğretmen adaylarının ispat şemalarının her çeşitini (dışsal, deneysel, analitik) kullandığını göstermektedir. Ayrıca sınıf seviyesi açısından öğretmen adaylarının kullandığı deneysel ve analitik ispat şemalarında anlamlı bir farklılık olduğu sonucuna ulaşılmıştır. Birinci sınıf matematik öğretmen adaylarının en çok deneysel ispat şemasını son sınıf öğretmen adaylarının ise daha çok analitik ispat şemasını kullandığı tespit edilmiştir. Araştırma sonunda elde edilen verilerden yola çıkılarak önerilerde bulunulmuştur.

***Anahtar Kelimeler*:** İspat, İspat Şemaları, Matematik Öğretmen Adayları

Introduction

Proofing has an important role in the process of comprehension and implementation of mathematics. Proof constitutes the most significant part of the components what makes mathematics itself (Padula, 2006). While Garnier and Taylor (1997) define proof as the effort for the imposition of a judgment or a conclusion's accuracy/fallacy by providing enough evidence, Hanna (2000) expresses that proof does not only indicates the accuracy/fallacy of a situation but also explains the reasons behind it. According to Harel ve Sowder (2007)

proofing is a mental action to annihilate the doubts of a society or an individual towards the accuracy of a claim.

Proof and proofing has a central importance in the processes of improving the ability to mathematically think, comprehension of the evolution and construction of mathematical knowledge, and perception of the level and usage of the phenomenon. Proof is accepted as one of the most significant concepts that constitutes the basics of mathematics and mathematics education (Ball and et al. 2002; Knuth, 2002; Lee, 2002; Padraig and McLoughlin, 2002; Uğurel and Moralı, 2010).

Ross (1998) defends that educational values related to proof should be taken into consideration more rather than the theoretical framework. He claims that gaining the logical ways of thinking will improve the evolution of proof and construction of knowledge in a different way. Therefore if reasoning ability of students will not be improved, mathematics would only mean following simple operations and imitating examples without conceptualizing them. Thus, the main aim of the mathematics education is to teach students how to generate meaningful solution ways, to support conceptualization abilities and logical thinking abilities consist of proper generalization and analyzation and synthesize (Dobos, Ocsko ve Vasarhelyi, 2001). These objectives demonstrate the importance of proof because of that proofing plays a substantial role in the process for generating these abilities (Terzi, Ünal ve Gürbüz, 2012).

The increasing importance of proof brought along many researches aiming to understand the processes of proofing of students from different age groups, thinking schemes during these processes, and changes in the way they approach to proofing. But results of these studies indicated that students from all education levels (primary, middle and high education) experiencing difficulties and proof is an action they do not like (Özer and Arıkan, 2002; Almedia, 2003; Jones, 2000; de Villiers, 1990; Raman, 2003 et al.. Moralı, Uğurel, Türnüklü and Yeşildere, 2006).

Generally proof strategies of students are insufficient (Weber, 2001), and they found meaningless the experiences in the process of proof (Galindo, 1998 akt. İskenderoğlu, 2010). Thus, most of the students prefer to memorize proof rather than understand it to pass the courses (Condradie and Firth, 2000). Because students do not face with proof adequately before undergraduate study (Knapp, 2005; Altun, Aşkar and Sarı, 2007), do not attend appropriate environments for the internalization and improvement of proof, they generate negative perspective towards proof.

Almeida (2000) claimed that the teachers' perception and experience related to proof affect students in the process to improve proof ability. Teachers should know the fundamentals of the knowledge that they teach to students and the answers of the potential questions in the minds of students to provide these abilities to students. Thus, it is expected from teachers to make their own conceptualization and interrogation. Therefore, pre-service mathematics teachers should be educated in a way to be qualified about proof (Güler and Dikici, 2012).

Proof should be integrated into the schedule of students starting from early years in accordance with their level and skills to improve logical thinking ability and to prevent problems related to proof. Even proof has a limited place in primary education it is the starting point of experiences related the proof, so that pre-service mathematics teachers' perspective about proof and their ability of proofing became more of an issue (Martin and Harel, 1989).

Proofs related to theory, rule or a problem plays an effective role in the processes of understanding the starting point of knowledge, comprehending relationships and analyzing the result. Understanding the fundamentals and concepts by visualizing the cause effect relationship can decrease the tendency to memorize. Thus, proofing needed to be emphasized. According to Hart (1994) scientific researches should be conducted related to students thinking processes to execute the reasons of mistakes in the proofing process (Weber, 2001).

Harel (2008) developed a theory that emphasizes the mental action, the ways of thinking and the ways of understanding and also the relationship among them. The theory which is named as DNR combines the concepts of Dualite, Necessity and Repeated Reasoning. These concepts mean that the instruction principles must base on the relationship between the ways of thinking and understanding, the mental necessity of students and internalization of information. Mathematics is considered as a set which consists of both the ways of thinking and the ways of understandings. According to Harel and Sowder (1998) the mathematics curriculum shouldn't be tackled only in terms of the ways of understanding it should also include the ways of thinking. Taking into consideration of this deficiency, they conducted a study about thinking processes that classifies solution levels during proofing by interrogating the reasons behind them. They grouped proof under three proof schemes; external, empirical and analytical. Proof schemes shows that how is persuaded and the way a person persuade others. On the other hand they are also important because they show students'thinking reactions during mathematical situations. Each proof scheme category is designed according to students' behaviors during proof processes and shows students' mental skills (Harel ve Sowder, 1998). Three proof schemes are explained below.

External Proof Schemes: Students using these schemes attributes the accuracy of their knowledge to books, rules or other people such as family, teacher (Flores, 2002; Flores, 2006). In this scheme students confirm their knowledge by supporting it with arguments that they learned before rather than reasoning. These schemes also consist of the proofs which were occurred without understanding the meanings of symbols and without comprehending the reasons algorithmic constructions (Harel and Sowder, 1998).

Empirical Proof Schemes: Students using these schemes give number values to expressions or use similar examples in the confirmation process, and they prefer to explain some situations with their intuitions (Harel and Sowder, 1998).

Analytical Proof Schemes: These schemes confirm assumptions with logical inferences and also include reasoning (Flores, 2002). These reasoning based on strong mathematical expression ability rather than trial and error method or examples. Students with these schemes use various strategies; do generalizations, and benefits mathematical relationships (Flores, 2002; Flores, 2006). According to Harel and Sowder (1998) analytical proof schemes constitutes the maximum level of proof.

From this point of view, this study is aimed to understand which proof schemes of Harel and Sowder (1998) are being used by pre-service mathematics teachers' to determine their proof level.

Methodology

Because the aim of the study is to indicate the actual situation descriptive research model is used.

Participants

Research group of study consists of 135 pre-service mathematics teachers (only freshman and seniors) who are having their undergraduate study at public university in 2011-2012 academic year.

Data Collection Tool

5 problems related to general knowledge in mathematics courses and designed according to the pre-service mathematics teacher's level are prepared to indicate the type of the schemes that they use in general mathematics course**.** 5 experts' opinions is asked about the measurement level to proof ability, representation level of general mathematics course and appropriateness level of class level of the problems. After taking their opinions, the convenience of problems is decided and implementation of them is carried out without any changes. Students are explained the aim of the study and are asked whether they are volunteer. 45 minutes is given to volunteer elementary pre-service mathematics teachers' to solve these problems and all solution ways in the papers are assessed.

Data analysis

In the analysis process, answers of elementary pre-service mathematics teachers are evaluated according to proof schemes' properties and classified into 4 categories; external, empirical, analytical, and empty. Percentage frequency tables of proof schemes prepared and bar chart is drawn. Also, it is analyzed that whether there is a significant difference between proof schemes according to class levels or not by using t test with a statistical programme and results are interpreted.

Findings

Findings about the proof schemes that freshman and senior pre-service mathematics teachers use in implementation are presented in the tables and interpreted.

Table 1. Percentage and Frequency Tables Related to Proof Schemes that Freshman and Senior Pre-service Mathematics Teachers Uses in Implementation

	Elementary Pre-service Mathematics Teachers Freshman		Elementary Pre-service Mathematics Teachers Senior		General	
Schemes	N	%	N	%	N	%
External	91	26.38	84	25.45	175	25.92
Emprical	150	43.47	42	12.73	192	28.44
Analytical	63	18.27	151	45.75	214	31.71
Empty	41	11.88	53	16.07	94	13.93
Total	345	100	330	100	675	100

Table 1, the percentage and frequency table indicates 675 answers of 135 pre-service mathematics teachers of to prepared problems (69 freshman 69x5=345 problem and 66 senior 66x5=330 problem). This table shows %25,92 of pre-service mathematics teachers use

external schemes, %28,44 use empirical schemes, %31,71 use analytical schemes, and %13,93 of them did not answer the problems. This data indicates that there are differences among proof schemes that pre-service mathematics teachers use.

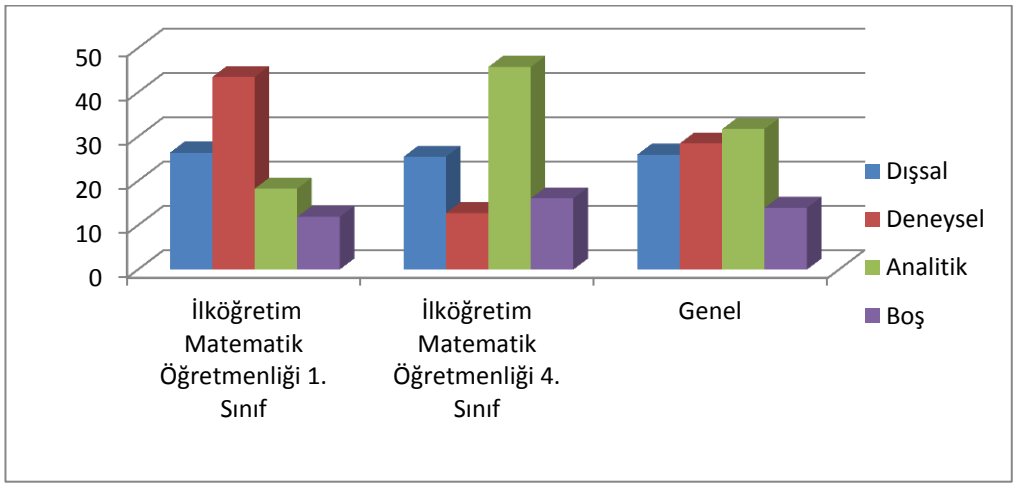

Figure 1: Bar Chart Related to Proof Schemes

Figure 1, shows the bar chart of percentage values of proof schemes used by elementary pre-service mathematics teachers according to class level. According to results of the study freshman of elementary pre-service mathematics teachers mostly use %43.47 (150 problems) empirical schemes, also %26.38 (91 problems) use external schemes, %18.27 (63 problem) use analytical schemes, %11.88'ini (41 problems) did not answer the problems. Senior students of elementary pre-service mathematics teachers mostly use %45.75'i (151 problems) analytical schemes, %25.45'i (84 problems) external schemes, %12.73'ü (42 problems) use empirical schemes, and %16.07'si (53 problems) did not answer the problems.

The findings presented in Table 1 indicate that freshmen mostly use empirical schemes and seniors mostly use analytical schemes. Besides, the usage of external schemes and the percentage of empty questions present similarities (very close) between freshman and senior elementary pre-service mathematics teachers. Even there is no significant difference between the usage percentages of proof schemes; the mostly used one is the analytical scheme, the second one is empirical, the third one is external schemes. Thus, the mostly used schemes are analytical schemes and the least used schemes are external schemes.

Table 2. T test Table related to Proof Schemes Freshman and Senior Pre-service Mathematics Teachers Uses in Implementation

Schemes	Classes	N	$\bar{X}$	S	Sd	T	P
External	Elementary Freshman	69	1.318	.848		-.821	.756
	Elementary Senior	66	1.272	.869	132.381		
	Total	135					
Emprical	Elementary Freshman	69	2.173	.890		6.474	.000
	Elementary Senior	66	.636	.905	132.501		
	Total	135					

Analytical	Elementary Freshman	69	.913	.903		-6.458	.000
	Elementary Senior	66	2.287	.872	132.985		
	Total	135					
Empty	Elementary Freshman	69	.594	.753		.396	.133
	Elementary Senior	66	.803	.845	129.741		
	Total	135					

Table 2 indicates the results of T test whether the differences between proof schemes that freshman and senior pre-service mathematics teachers use in implementation is significant or not. According to values in the table, there is a significant difference ($p < .05$) between proof schemes that freshman and senior pre-service mathematics teachers use in terms of empirical and analytical schemes. Besides there is no significant relationship between the proof schemes in terms of external schemes according to class level.

When the averages in the table taken into consideration it is seen that freshman use empirical schemes more than seniors, while they use analytical schemes less than seniors. Also, there is no significant relationship ($p < .05$) between class level and empty questions

Discussion and Conclusion

Elementary pre-service mathematics teachers used all proof schemes (empirical, analytical and external) in the solutions of the 5 problems during the study. Thus, our results are in the same direction with other studies conducted before (Sowder ve Harel, 1998; Flores, 2002; İskenderoğlu, 2003; Housman and Porter, 2003; Stylianou, 2006; Flores, 2006; Baki and et al., 2009; İskenderoğlu, 2010).

It is concluded that the mostly used proof schemes by elementary pre-service mathematics teachers is analytical schemes and the least used proof schemes is external schemes. But there are some other studies concluded that the mostly used proof schemes are empirical and external schemes among students (Harel and Sowder, 1998; Harel and Sowder, 2003). This difference may be derived from the tendency of the education faculty members towards analytical schemes, their usage of analytical schemes in their courses for the solution of the problems, and its reflection on students.

Also there is a significant difference found between the usage of empirical and analytical proof schemes in terms of class level. On the other hand, it is concluded that there is no significant relationship between the usage of external proof schemes and empty questions in terms of class level. Results indicated that freshman elementary pre-service mathematics teachers mostly use empirical proof schemes and senior elementary pre-service mathematics teachers mostly prefer analytical proof schemes. There are studies claiming that the empirical schemes are the commonly used proof schemes (Coe and Ruthven, 1994; Galbraith, 1995;

Galindo, 1998; Harel and Sowder, 1998; Harel and Sowder, 2003; Knapp and Zandieh, 2004; Rodriguez, 2006; İskenderoğlu, 2010). Özer and Arıkan (2002) emphasized that high school students have propensity to use assign method or induction in ht proof process. Because freshman did not face with proof commonly before undergraduate study, freshman elementary pre-service mathematics teachers may have used empirical schemes as a result of their habits

form high school. Also, the heavy usage of analytical schemes by seniors can be explained with the effect of the courses at university and the reflection of the implemented schedule. İskenderoğlu (2010) concluded that the empirical schemes are mostly used by freshman elementary pre-service mathematics teachers and analytical schemes are mostly used by senior elementary pre-service mathematics teachers. Results of our study are parallel with his study.

This study indicates that students use different proof schemes, have different reasoning methods and approach same questions with their own strategy while proofing. This finding of the study is supporting the expression of Raman (2003) "Proof can include different strategies and can be made more than one way". It is possible to claim that the differences between the reasoning methods and perspectives will bring innovation into mathematics.

In conclusion, teachers and pre-service teachers have important responsibilities in the way to improve thinking abilities of the students. Different ways of solutions should be provided in class activities and questions requires different reasoning techniques should be asked in class activities to pre-service mathematics teachers. Also teachers should provide an environment that allows pre-service mathematics teachers to visualize origin and reasons of concepts. Thus, the innovative and correct construction of concepts and thinking methods of pre-service teachers can be improved. Besides some of the pre-service teachers believe that proof is unnecessary and they do not want to make proof. Activities to explain the necessity of proof and indicate the usage levels of it should be provided. Also, proper examples should be selected to internalize the concepts and algorithmic structure, to explain the proof without memorizing, and to break their high school habits. The sample of the research includes only the freshman and senior elementary pre-service mathematics teachers. This can be a limitation and another implementation of this study with a greater sample and with different problems can be suggested.

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