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PROPOSAL OF A SOLUTION TO MULTI OBJECTIVE LINEAR FRACTIONAL PROGRAMMING PROBLEM

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ABSTRACT

In this paper, we have proposed a solution algorithm for solving Multi Objective Linear Fractional Programming Problem (MOLFPP). We have positive objective functions for all $x \in X$ by translating objective functions that are negative for some $x \in X$. Then, we have a single objective function as minimization of the deviations from maximum values of objective functions in the feasible region . So, MOLFPP transformed to goal programming problem. The proposed algorithm to solving MOLFPP always yields efficient solutions.
Using the proposed solution to MOLFPP have been given three numerical examples and solved them using computer packet program WinQSB.
Keywords: Multi Objective Linear Fractional Programming Problem (MOLFPP), Goal Programming Method, WinQSB.

ÇOK AMAÇLI LİNEER KESİRLİ PROGRAMLAMA PROBLEMİNE BİR ÇÖZÜM ÖNERİSİ

ÖZET

Bu makalede, Çok Amaçlı Lineer Kesirli Programlama Problemine bir çözüm algoritması önerdik. Uygun çözüm bölgesindeki bazı $x \in X$ değerleri için negatif değerlere sahip amaç fonksiyonlarına dönüşüm uygulayarak tüm amaç fonksiyonlarını uygun bölgede pozitif yaptık. Daha sonra, amaç fonksiyonlarının uygun bölgedeki maksimum değerlerinden sapmalarını minimize ederek, aynı uygun bölgede Çok Amaçlı Lineer Kesirli Programlama Problemini tek amaçlı lineer programlama problemine, yani, hedef programlamaya dönüştürdük. Elde edilen hedef programlama probleminin optimal çözümü daima Çok Amaçlı Lineer Kesirli Programlama Probleminin etkin çözümlerini vermektedir.
Önerdiğimiz çözüm algoritmasını kullanarak üç farklı sayısal örnek verilmiştir.Verilen örnekler WinQSB paket programıyla çözülmüştür..
Anahtar Sözcükler: Çok Amaçlı Lineer Kesirli Programlama Problemi, Hedef Programlama, WinQSB.

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1. INTRODUCTION

In real word applications like financial and corporate planning, production planning, marketing and media selection, university planning and student admissions, health care and hospital planning, ore blending problems, court system, air force maintenance units, bank branches, etc. frequently may be faced up with decision to optimise dept/equity ratio, profit/cost, inventory/sales, actual cost/standard cost, output/employee, student/cost, nurse/patient ratio etc. respect to some constraints [1].

In literature, different approaches appear to solve different models of linear fractional programming problem (LFPP). Because, fractional programming solves more efficiently the above problems respect to Linear Programming (LP).

When some of the studies have achieved solution methods [1-3, 7-9], others have concentrated on applications [1, 4-7]. In these studies are discussed in details LFP. It is showed that a linear fractional programming problem (LFPP) can be optimised easily. But, in the great scale decision problems, there is more than one objective, which must be satisfied at the same time as possible. However, most of these are linear fractional objectives. It is difficult to talk about the optimal solutions of these problems. The solutions searched for these problems are weak efficient or strong efficient. If required one compromise solution can be reached by the affection of the models with the decision makers (DMs).

There exist several methodologies to solve multi objectives linear fractional programming problem (MOLFPP) in the literature. Most of these methodologies are computationally burdensome [7]. Kornbluth and Steuer, Y.J.Lai and C.L.Hwang have developed an algorithm for solving the MOLFPP for all weak-efficient vertices of the feasible region [1,11]. I. Nykowski, Z.Zolkiewski have proposed a compromise procedure for MOLFPP[12]. Choo and Atkins have given an analysis of the bicriteria LFP [10].

M.K.Luhandjula solved MOLFPP using a fuzzy approach [1,13]. He used linguistic approach to solve MOLFPP by introducing linguistic variables to represent linguistic aspirations of the DM. Dutta et. modified the linguistic approach of Luhandjula to solve MOLFPP [13,14].

In this paper, we have proposed an algorithm solution for solving Multi Objective Linear Fractional Programming Problem (MOLFPP). We have positive objective functions for all $x \in X$ by translating objective functions that are negative for some $x \in X$. Then, we have a single objective function as minimization of the deviations from maximum values of objective functions in the feasible region. So, MOLFPP transformed to goal programming problem. The proposed algorithm solution to MOLFPP always yields efficient solutions.

2. LINEAR FRACTIONAL PROGRAMMING (LFP)

The general format of Linear Fractional Programming may be written as

$$Max \frac{c^T x + \alpha}{d^T x + \beta}$$

$$\begin{aligned} \text{s.t} \quad & Ax = b, \\ & x \geq 0, \quad x, c^T, d^T \in R^n \\ & A \in R^{mxn}, \alpha \ \beta \in R, d^T x + \beta > 0. \end{aligned} \tag{1}$$

or

$$Max \frac{N(x)}{D(x)}$$

s.t. $Ax \le b,$

$$x \ge 0 \quad and \quad x \in X = \{x : \ Ax \le b, \ \ x \ge 0\} \Rightarrow D(x) > 0.$$

Definition 1. (Craven):
The two Mathematical programming problems
(i) Max $F(x)$ subject to $x \in X$,
(ii) Max $G(x)$, subject to $x \in X$, will be said to be equivalent if only if is a one one map q(.) of the feasible set of (i), onto the feasible set of (ii), such that *F(x)=G(q(x))* for all $x \in X$ [7].

3. A PROPOSED SOLUTION ALGORITHM OF THE MULTIPLE OBJECTIVE LINEAR FRACTIONAL PROGRAMMING PROBLEM (MOLFPP)

A MOLFPP may be written following as;

$$Max\ Z(x)=\{Z_1(x), Z_2(x),..., Z_k(x)\} \tag{2}$$

$$x \in X = \{x \in R^n; \quad Ax \le b, \quad x \ge 0\},$$

$$with \quad b \in R^n, \quad A \in R^{mxn}$$

and
$$Z_i(x) = \frac{c_i x + \alpha_i}{d_i x + \beta_i} = \frac{N_i(x)}{D_i(x)}, \quad c_i, d_i \in R^n and\ \alpha_i, \beta_i \in R. \tag{3}$$

Let x_i^* be a global maximum point of $Max \quad Z_i(x) = \dfrac{c_i x + \alpha_i}{d_i x + \beta_i}$, $\forall i$.

The solution algorithm for MOLFPP is shown below.

Step 1. Find values of objective functions, $Z^* = \{Z_1^*, Z_2^*, ..., Z_k^*\}$, that maximizes objective functions $Max \quad Z_i(x) = \dfrac{c_i x + \alpha_i}{d_i x + \beta_i}$, for all i at feasible region,

$$x \in X = \{x \in R^n; Ax \le b, x \ge 0, b \in R, A \in R^{mxn}\}.$$

Step 2. Calculate to $Z_i^{Min} = Min\left\{\dfrac{c_i x + \alpha_i}{d_i x + \beta_i}\right\}$, If Z_i^* <0, for some i and do positive $Z_i(x) \ge 0$ using transformation $Z_i(x) = Z_i(x) - Z_i^{Min}(x)$ for some i which is Z_i^* <0 and then calculate new maximum objective values, $Z_i^* = Z_i^* - Z_i^{Min}$ for some i which is Z_i^* <0.

Step 3. Get to
$$\frac{c_i x + \alpha_i}{d_i x + \beta_i} \le \frac{c_i x_i^* + \alpha_i}{d_i x_i^* + \beta_i}, \qquad \text{for i=1,2,…,k.} \tag{4}$$

where $\dfrac{c_i x^* + \alpha_i}{d_i x^* + \beta_i}$ is quantitative value of the ith linear fractional objective function.

Step 4. Write as $c_i x+\alpha_i - \dfrac{c_i x^* +\alpha_i}{d_i x^* +\beta_i}(\ d_i x+\beta_i\ )\le 0$ since $d_i x + \beta_i\ > 0$.

Step 5. In order to satisfy these targets in step (4), the goal programming model for MOLFPP is proposed as follows:

$$(c_i x+\alpha_i)-\frac{c_i x^* +\alpha_i}{d_i x^* +\beta_i}(d_i x+\beta_i)+h_i^- = 0 \qquad \text{for i=1,2…k,} \tag{5}$$

where h_i^- is a negative deviation from the ith goal $\dfrac{c_i x^* +\alpha_i}{d_i x^* +\beta_i}(d_i x+\beta_i)$.

Since the linear fractional objectives in problem (1) should be maximized, total negativedeviation from goals, $\sum_{i=1}^{k} h_i^- \ge 0$ is should be minimized. Let $s = \sum_{i=1}^{k} h_i^-$. Then, model with a single objective function for MOLFPP may be written as the minimization of the total deviations from the above stated goals as follow:

$$s = \underset{x\in X}{Min} \sum_{i=1}^{k} h_i^-$$

Subject to

$$(c_i x+\alpha_i)-\frac{c_i x^* +\alpha_i}{d_i x^* +\beta_i}(d_i x+\beta_i)+h_i^- = 0 \qquad \text{for i=1,2…k.} \tag{6}$$

Step 6. Find optimal solution of the above linear programming problem (6)

In Problem (6), set X is non-empty convex set having feasible points. In fact; the solution of problem (6) gives the efficient solution of problem (1).

5. NUMERICAL EXAMPLES

Example 1. Let us consider a MOLFPP with two objectives as follows:

$$Max\left\{ Z_1(x) = \frac{-3x_1+2x_2}{x_1+x_2+3},\ Z_2(x) = \frac{7x_1+x_2}{5x_1+2x_2+1} \right\}$$

s.t. $x_1 - x_2 \ge 1$

$2x_1 + 3x_2 \le 15$

$x_1 \ge 3$

$x_1, x_2 \ge 0.$

It is observed that $Z_1 < 0, Z_2 \ge 0,$ for each x in the feasible region [7].

$$\frac{-15}{7} \le Z_1 \le \frac{-14}{23}, \qquad \frac{139}{121} \le Z_2 \le \frac{105}{77},$$

$Z_1(x) = \dfrac{-3x_1+2x_2}{x_1+x_2+3}$ becomes $Z_1(x) = \dfrac{-6x_1+29x_2+45}{7(x_1+x_2+3)}$ by transformation $Z_i(x) = Z_i(x) - Z_i^{Min}(x)$ and $0 \le Z_1 \le \dfrac{247}{161}$.

$$\frac{-6x_1 + 29x_2 + 45}{7(x_1 + x_2 + 3)} \le \frac{247}{161}$$

$$\frac{7x_1 + x_2}{5x_1 + 2x_2 + 1} \le \frac{105}{77},$$

Thus, we have

$$16.739x_1 - 18.26x_2 - h_1^- = 12.78$$

$$0.1818x_1 - 1.7272x_2 + h_2^- = 1.3636$$

The given MOLFP reduces to following goal programming problem,

$s = Min \quad (h_1^- + h_2^-)$

s.t

$$16.739x_1 - 18.26x_2 - h_1^- = 12.78$$

$$0.1818x_1 - 1.7272x_2 + h_2^- = 1.3636$$

$$x_1 - x_2 \ge 1$$

$$2x_1 + 3x_2 \le 15$$

$$x_1 \ge 3$$

$$x_1, x_2 \ge 0.$$

The solution to the above goal program is obtained as $x_1 = 3, \ x_2 = 2, \ h_1^- = 0.917, h_2^- = 4.2726, \min s = 5.1896$. The original problem is obtained as

$$x_1 = 3, \quad x_2 = 2, \quad Z_1 = \frac{-5}{8}, \quad Z_2 = \frac{23}{20}.$$

Example 2. Let us consider a MOLFPP with three objectives as follows:

$$\text{Max } \left\{ Z_1(x) = \frac{-3x_1 + 2x_2}{x_1 + x_2 + 3}, Z_2(x) = \frac{7x_1 + x_2}{5x_1 + 2x_2 + 1}, Z_3(x) = \frac{x_1 + 4x_2}{2x_1 + 3x_2 + 2} \right\}$$

s.t. $x_1 - x_2 \ge 1$

$2x_1 + 3x_2 \le 15$

$x_1 + 9x_2 \ge 9$

$x_1 \ge 3$

$x_1, x_2 \ge 0.$

It is observed that $Z_1 < 0, Z_2 \ge 0, Z_3 \ge 0$ for each x in the feasible region [7].

$$\frac{-53}{26} \le Z_1 \le \frac{-14}{23}, \quad \frac{139}{121} \le Z_2 \le \frac{23}{17}, \quad \frac{8}{17} \le Z_3 \le \frac{14}{17}.$$

Because of $Z_1 < 0$, $Z_1(x)$ becomes $Z_1(x) = \dfrac{-25x_1 + 105x_2 + 159}{26x_1 + 26x_2 + 78}$ and $0 \le Z_1 \le \dfrac{855}{598}$.

$$\frac{-3x_1 + 2x_2}{x_1 + x_2 + 3} \le \frac{-14}{23},$$

$$\frac{7x_1 + x_2}{5x_1 + 2x_2 + 1} \le \frac{23}{17},$$

$$\frac{x_1 + 4x_2}{2x_1 + 3x_2 + 2} \le \frac{14}{17}$$

$$\frac{-25x_1 + 105x_2 + 159}{26x_1 + 26x_2 + 78} \le \frac{855}{598},$$

$$\frac{7x_1 + x_2}{5x_1 + 2x_2 + 1} \le \frac{23}{17},$$

$$\frac{x_1 + 4x_2}{2x_1 + 3x_2 + 2} \le \frac{14}{17},$$

Thus, we have

$$2.3917x_1 - 2.60824x_2 \ge 1.82612,$$
$$0.2354x_1 - 1.7059x_2 \le 1.3529,$$
$$-0.647x_1 + 1.5294x_2 \le 1.647.$$

The given MOLFP problem is written as

$$s = Min(h_1^- + h_2^- + h_3^-)$$

s.t.

$$2.3917x_1 - 2.60824x_2 - h_1^- = 1.82612,$$
$$0.2354x_1 - 1.7059x_2 + h_2^- = 1.3529,$$
$$-0.647x_1 + 1.5294x_2 + h_3^- = 1.647.$$

$$x_1 - x_2 \ge 1$$
$$2x_1 + 3x_2 \le 15$$
$$x_1 + 9x_2 \ge 9$$
$$x_1 \ge 3$$
$$x_1, x_2 \ge 0.$$

The solution to the above LP is obtained as $Min\, s = 4.7204$, $x_1 = 3$, $x_2 = 2$, $h_1^- = 0.1327$, $h_2^- = 4.0585$, $h_3^- = 0.5292$. The solution for original problem is obtained as

$x_1 = 3, x_2 = 2,\ Z_1 = \frac{-5}{8}, Z_2 = \frac{23}{20}, Z_3 = \frac{11}{14}.$

Example 3. Let us consider a MOLFPP with three objectives that all objectives are negative as follows:

$$\text{Max } \left\{ Z_1(x) = \frac{-x_1 + x_2 - 4}{6x_1 + x_2 + 3},\ Z_2(x) = \frac{x_1 - x_2 - 5}{x_2 + 1}, Z_3(x) = \frac{3x_1 + x_2 - 17}{-3x_1 + 16} \right\}$$

s.t.

$x_1 \le 4$

$x_2 \le 4$

$x_1 + x_2 \le 7$

$-x_1 + x_2 \le 3$

$x_1 - x_2 \le 3$

$x_1, x_2 \ge 0.$

It is observed that $Z_1 < 0, Z_2 < 0, Z_3 < 0$ for each x in the feasible region.

$\frac{-4}{3} \le Z_1 \le \frac{-1}{13}, \quad -5 \le Z_2 \le -1, \quad -\frac{8}{7} \le Z_3 \le -\frac{1}{2}.$

Since all objective functions , $Z_1 < 0, Z_2 < 0, Z_3 < 0$ for each x in the feasible region,

These new objective function are $Z_1(x) = \frac{21x_1 + 7x_2}{3(6x_1 + x_2 + 3)}$, $Z_2(x) = \frac{x_1 + 4x_2}{x_2 + 1}$, $Z_1(x) = \frac{-3x_1 + 7x_2 + 9}{7(-3x_1 + 16)}$ and these maximum values are $0 \le Z_1 \le \frac{49}{39}$, $0 \le Z_2 \le 4$, $0 \le Z_3 \le \frac{9}{14}$.

Hence $-3x_1 + 6x_2 \le 21$, $x_1 \le 4$, $21x_1 + 14x_2 \le 126$

The given MOLFP problem is written as

$s = Min(h_1^- + h_2^- + h_3^-)$

s.t.

$-3x_1 + 6x_2 + h_1^- = 21$,

$x_1 + h_2^- = 4$

$21x_1 + 14x_2 + h_3^- = 126$

$x_1 \le 4$

$x_2 \le 4$

$x_1 + x_2 \le 7$

$-x_1 + x_2 \le 3$

$x_1 - x_2 \le 3$

$s, x_1, x_2 \ge 0.$

The solution to the above LP is obtained as Min $s = 13$, $x_1 = 3$, $x_2 = 4$, $h_1^- = 6$, $h_2^- = 0$, $h_3^- = 7$.

The solution for original problem is obtained as $x_1 = 3, x_2 = 4,\ Z_1 = \frac{-3}{25}, Z_2 = -\frac{6}{5}, Z_3 = -\frac{4}{7}$.

6. CONCLUSION

In this paper, we have proposed an algorithm for solving Multi Objective Linear Fractional Programming Problem (MOLFPP). We have positive objective functions for all $x \in X$ by translating objective functions that are negative for some $x \in X$. Then, we have a single objective function as minimization of the deviations from maximum values of objective functions in the feasible region. So, MOLFPP transformed to goal programming problem. The proposed solution algorithm to MOLFPP always yields efficient solutions.

Using the proposed solution algorithm to MOLFPP have been given three numerical examples and solved them using computer packet program WinQSB.

REFERENCES

[1] Y.J.Lai,C.L.Hwang, Fuzzy Multiple Objective Decision Making, Springer,1996.

[2] A.Charnes, W.W. Cooper, programming with linear fractional functionals, Nav. Res. Logistics Quart. 9 (1962), 181-186.

[3] S.Zionts, Programming with linear fractional functionals, Nav. Res. Logistics Quart. 15 (1968), 449-451.

[4] E.Munteanu, I. Rado, Calculul, Sarjelar celor mai economice la cuptoarecle de topit fonta, studii si cercetari matematice , cluj, faseiola anexa XI, 1960, pp 149- 158.

[5] A.C. Gilmore , R.E. Gomory, A linear programming approach to the cutting stock problem II, Oper. Res 11 (1963) 863-888.

[6] A.Sengupta,T.P.Pal,M.Chakraborty,Interpretation of inequality constraints involving interval coefficients and a solution to interval linear programming, Fuzzy Sets and systems 119(2001)129-138.

[7] M.Chakraborty, Sandipan Gupta, Fuzzy mathematical programming for multi objective linear fractional programming problem, Fuzzy Sets and Systems 125 (2002) 335-342.

[8] S.Schaible, Analyse and Anwendungen von Quatienten programmen verlag Anton Hain, Meisenheim am Glan , 1978.

[9] B.D.Craven, Fractional Programming, Heldermann Verlag, Berlin, 1988.

[10] E.U. Choo, D.R. Atkins, Bicriteria Linear fractional programming, J.Optim. Theory Appl. 36 (1982), 203-220.

[11] J.S.H. Kornbluth, R.E. Steuer, Multiple Objective Linear Fractional Programming Manage . Sci. 27 (1981) 1024-1039.

[12] I. Nykowski, Z.Zolkiewski, A compromise procedure for the multiple objective linear fractional programming problem, European J. Oper. Res. 19 (1985) 91-97.

[13] M.K. Luhandjula, Fuzzy approaches for multiple objective linear fractional optimization, Fuzzy Sets and Systems 13, (1984) 11-23.

[14] D.Dutta, R.N. Tiwari, J.R.Rao, Multiple objective linear fractional programming -A fuzzy set theoretic approach, Fuzzy Sets and Systems 52 (1992) 39-45.