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FREE VIBRATION ANALYSIS OF ELASTICALLY SUPPORTED TIMOSHENKO BEAMS

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ABSTRACT

In this study, free vibration of elastically supported beams is investigated based on Timoshenko beam theory (TBT). The Lagrange equations are used to examine the free vibration characteristics of Timoshenko beams. In the study, for applying the Lagrange equations, trial functions denoting the deflection and the rotation of the the cross-section of the beam are expressed in the power series form. By using the Lagrange equations, the problem is reduced to the solution of a system of algebraic equations. The influence of stiffnesses of the supports on the free vibration characteristics of Timoshenko beams is investigated. For this purpose, the first three eigenvalues of the Timoshenko beams are calculated for various rigidity values of translational and rotational springs, and obtained results are not only tabulated, but also presented in three-dimensional plots. It is thought that the tabulated results will prove useful to designers and provide a reference against which other researchers can compare their results.
Keywords: Timoshenko beam theory, Lagrange equations, Power series.
MSC number/numarası: 53A40, 74H45.

ELASTİK MESNETLİ TİMOSHENKO KİRİŞLERİNİN SERBEST TİTREŞİMLERİNİN İNCELENMESİ

ÖZET

Bu çalışmada elastik mesnetli kirişlerin serbest titreşimleri Timoshenko kiriş teorisi (TBT) çerçevesinde incelenmiştir. Problemin çözümü için Lagrange denklemleri kullanılmıştır. Lagrange denklemlerinin uygulanması için kirişin düşey yerdeğiştirmelerini ve kiriş kesitlerinin dönmelerini ifade eden çözüm fonksiyonlarının oluşturulmasında kuvvet serileri kullanılmıştır. Lagrange denklemleri kullanılarak problem cebrik denklem sisteminin çözümüne indirgenmiştir. Mesnet rijitliklerinin Timoshenko kirişlerinin serbest titreşimleri üzerindeki etkisi araştırılmıştır. Bu amaçla, elastik mesnetli Timoshenko kirişlerinin ilk üç özdeğeri, dönme ve çökmeye karşı elastik yaylarının farklı rijitlik değerleri için elde edilmiş ve elde edilen sonuçlar hem tablo hem de üç boyutlu grafikler halinde verilmiştir. Tablolaştırılan sonuçların tasarımcılar için faydalı olacağı ve diğer araştırmacıların sonuçlarını karşılaştırmada referans oluşturabileceği düşünülmektedir.
Anahtar Sözcükler: Timoshenko kiriş teorisi, Lagrange denklemleri, Kuvvet serileri.

1. INTRODUCTION

Many studies have been carried out related with the problem of free vibration of beams with elastically supports based on the Euler-Bernoulli beam theory (EBT) or Timoshenko beam theory

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(TBT). The well-known Euler-Bernoulli beam theory (EBT) states that plane sections remain plane and perpendicular to the central axis of the beam after deformation, regarding transverse shear strain to be neglected. Although this theory is very useful for slender beams and columns, it does not give accurate solutions for thick beams. In the Timoshenko beam theory (TBT), the normality assumption of the Euler-Bernoulli theory (EBT) is relaxed and a constant state of transverse shear strain with respect to the thickness coordinate is included. The Timoshenko beam theory requires shear correction factors to compensate for the error due to this constant shear stress assumption.

The problem of free lateral vibration of an axially loaded Euler-Bernoulli beam with intermediate elastic supports and concentrated masses is considered using Green function method by Kukla [2]. H. K. Kim and M. S. Kim [3] presented a method to find accurate vibration frequencies of beams with elastic supports using Fourier series. Nallim and Grossi [4] presented a simple variational approach based on the use of the Rayleigh-Ritz method with the characteristic orthogonal polynomial shape functions for the determination of free vibration frequencies of beams with several complicating effects within the frame of Euler-Bernoulli beam theory. Lee and Schultz [5] applied the pseudospectral method to the eigenvalue analysis of Timoshenko beams. Zhou [6] used the Rayleigh-Ritz method for the free vibration of multi-span Timoshenko beams. Farghaly [7] has investigated the natural frequencies and the critical buckling load coefficients for a multi-span Timoshenko beam. Banerjee [8] investigated the free vibration analysis of axially loaded Timoshenko beams by using the dynamic stiffness method. The free vibration of Timoshenko beams with internal hinge and subjected to axial tensile load is carried out by Lee et. al. [9]. A dynamic investigation method for the analysis of Timoshenko beams which takes into acount shear deformation is proposed by Auciello and Ercolano [10]. In [10], the solution of the problem is obtained through the iterative variational Rayleigh-Ritz method. The free vibration of Timoshenko beams having classical boundary conditions, which was satisfied by Lagrange multipliers, was investigated for different thickness-to-length ratios by Kocatürt and Şimşek [11].

In the present study, the free vibration of elastically supported Timoshenko beams is analyzed by using the Lagrange equations with the trial functions in the power series form denoting the deflection and the rotation of the cross-section of the beam. The convergence study is based on the numerical values obtained for various numbers of power series terms. In the numerical examples, the first three eigenvalues of the Timoshenko beam are calculated for various values of stiffness of translational and rotational springs. The accuracy of the results is established by comparison with previously published accurate results for the free vibration analysis of the Timoshenko beams.

2. THEORY AND FORMULATIONS

Consider a straight uniform single-span Timoshenko beam of length L, depth h, width b, having rectangular cross-section as shown in Fig. 1, where K_i and R_i are the translational and rotational spring constant. A Cartesian coordinate system (X, Y, Z) is defined on the central axis of the beam, where the X axis is taken along the central axis, with the Y axis in the width direction and the Z axis in the depth direction. Also, the origin of the coordinate system is chosen at the mid-point of the total length of the beam.

Although, it is possible to take lots of point supports at arbitrary points, in the numerical investigations here, it will be considered that the beam is supported at the two end points, where the parameters K_i and R_i are taken to have the same values at all the supports denoted by $K_i = K$ and $R_i = R$.

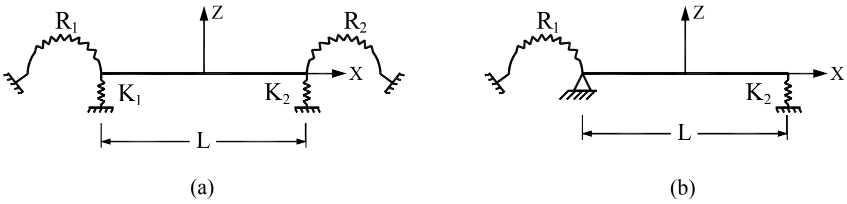

Figure 1. Considered Timoshenko beams with (a) the first type and (b) the second type of translational and rotational springs.

The Timoshenko beam theory is based on the following displacement fields;

$$U(X,Z,t) = -Z\,\Phi(X,t)$$
$$W(X,Z,t) = W(X,t)\,, \tag{1}$$

where $W(X,t)$ is the transverse displacement of a point on the beam reference plane and $\Phi(X,t)$ is the rotation of a normal to the reference plane about y-axis.

According to the Timoshenko beam theory (TBT), the elastic strain energy of the beam at any instant is expressed as an integral in Cartesian coordinates as follows

$$U = \frac{1}{2}\int_{-\frac{L}{2}}^{\frac{L}{2}} \left[EI(x)\left(\frac{d\Phi(X,t)}{dX}\right)^2 + k_s GA(x)\left(\frac{dW(X,t)}{dX} - \Phi(X,t)\right)^2 \right] dX\,, \tag{2}$$

where E is the Young's modulus, G is the transverse shear modulus, $I(X)$ is the moment of inertia, $A(X)$ is the area of the cross-section and k_s is a constant that accounts for non-uniform shear stress distribution through the thickness.

Kinetic energy of the beam at any instant is

$$K_e = \frac{1}{2}\int_{-\frac{L}{2}}^{\frac{L}{2}} \left[\rho A(X)\left(\frac{dW(X,t)}{dt}\right)^2 + \rho I(X)\left(\frac{d\Phi(X,t)}{dt}\right)^2 \right] dX\,, \tag{3}$$

where ρ is the mass of the beam per unit volume.

Additive strain energy of the translational and rotational spring is given by the following formulas, respectively

$$F_T = \frac{1}{2}\sum_{i=1}^{2} K_i \left[W(X_{Si},t)\right]^2 \tag{4a}$$

$$F_R = \frac{1}{2}\sum_{i=1}^{2} R_i \left[\Phi(X_{Si},t)\right]^2\,, \tag{4b}$$

where X_{S_i} denotes the location of the i th support, K_i and R_i are the spring constant of the translational and rotational springs at the both ends, respectively.

By introducing the following non-dimensional parameters

$$x = \frac{X}{L},\ \bar{w} = \frac{W}{L},\ \bar{\phi} = \Phi\,, \tag{5}$$

the potential and kinetic energy of the beam at any instant can be written as

$$U = \frac{1}{2}\int_{-1/2}^{1/2}\left[\frac{E\,I(x)}{L}\left(\frac{d\,\overline{\phi}(x,t)}{d\,x}\right)^2 + k_s G\,L\,A(x)\left(\frac{d\,\overline{w}(x,t)}{d\,x} - \overline{\phi}(x,t)\right)^2\right]d\,x \tag{6}$$

$$K_e = \frac{1}{2}\int_{-1/2}^{1/2}\left[\rho\,L^3 A(x)\left(\frac{d\,\overline{w}(x,t)}{d\,t}\right)^2 + \rho\,L\,I(x)\left(\frac{d\,\overline{\phi}(x,t)}{d\,t}\right)^2\right]d\,x\,. \tag{7}$$

Additive strain energy of the translational and rotational springs can be written in terms of the non-dimensional quantities in the following equations:

$$F_T = \frac{L^2}{2}\sum_{i=1}^{2} K_i\,[\overline{w}(x_{Si},t)]^2 \tag{8}$$

$$F_R = \frac{1}{2}\sum_{i=1}^{2} R_i\,[\overline{\phi}\,(x_{Si},t)]^2\,. \tag{9}$$

It is known that some expressions satisfying geometrical boundary conditions are chosen for $\overline{w}(x,t)$ and $\overline{\phi}(x,t)$, and by using the Lagrange equations, the natural boundary conditions are also satisfied. Therefore, by using the Lagrange equations and by representing the transverse displacement $\overline{w}(x,t)$ and the rotation of cross-sections $\overline{\phi}(x,t)$ in terms of a series of admissible functions and adjusting the coefficients in the series to satisfy the Lagrange equations, approximate solutions are found for the displacement and the rotation functions. For applying the Lagrange equations, the trial functions $\overline{w}(x,t)$ and $\overline{\phi}(x,t)$ are approximated by space-dependent polynomial terms $x^0, x^1, x^2, ..., x^{M-1}$ and time-dependent displacement coordinates $\overline{a}_m(t)$ and $\overline{b}_m(t)$. Thus

$$\overline{w}(x,t) = \sum_{m=1}^{M} \overline{a}_m(t)\,x^{m-1} \tag{10}$$

$$\overline{\phi}(x,t) = \sum_{m=1}^{M} \overline{b}_m(t)\,x^{m-1}\,. \tag{11}$$

The time-dependent generalized coordinates for the free vibration of the beam can be expressed as follows:

$$\overline{a}_m(t) = a_m\,e^{i\,\omega\,t} \tag{12}$$

$$\overline{b}_m(t) = b_m\,e^{i\,\omega\,t}\,. \tag{13}$$

Dimensionless amplitudes of the displacement and normal rotation of a cross-section of the beam can be expressed as follows:

$$w(x) = \sum_{m=1}^{M} a_m\,x^{m-1}\,, \tag{14}$$

$$\phi(x) = \sum_{m=1}^{M} b_m\,x^{m-1} \tag{15}$$

The functional of the problem is

$$L = K_e - (U + V + F_T + F_R)\,. \tag{16}$$

The Lagrange equations are given as follows;

$$\frac{\partial L}{\partial q_k} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) = 0\,, \qquad k = 1, 2, ..., 2M \tag{17}$$

where the overdot stands for the partial derivative with respect to time and

$$q_k = a_k \qquad k = 1, 2, ..., M \tag{18}$$

$$q_k = b_k \qquad k = M+1, ..., 2M\,. \tag{19}$$

By introducing the following non-dimensional parameters

$$\lambda^2 = \frac{\rho A \omega^2 L^4}{EI},\ \beta = \frac{k_s G A L^2}{EI},\ \mu = \frac{I}{AL^2},\ \kappa_i = \frac{K_i L^3}{EI},\ \theta_i = \frac{R_i L}{EI} \tag{20}$$

and by using Eq. (17), the following simultaneous sets of linear algebraic equations are obtained which can be expressed in the following matrix forms;

$$\begin{bmatrix} [A] & [B] \\ [C] & [D] \end{bmatrix} \begin{Bmatrix} \{\bar{a}\} \\ \{\bar{b}\} \end{Bmatrix} - \lambda^2 \begin{bmatrix} [E] & [0] \\ [0] & [F] \end{bmatrix} \begin{Bmatrix} \{\bar{a}\} \\ \{\bar{b}\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \end{Bmatrix} \tag{21}$$

where $[A], [B], [C], [D], [E]$ and $[F]$ are the coefficient matrices obtained by using Eq. (17) and

$$A_{km} = \int_{-0\cdot5}^{0\cdot5} \beta\, (x^{k-1})'(x^{m-1})' d\,x + \kappa_1\, (-0\cdot5)^{k-1}(-0\cdot5)^{m-1} + \kappa_2\, (0\cdot5)^{k-1}(0\cdot5)^{m-1}$$

$$B_{km} = \int_{-0\cdot5}^{0\cdot5} \beta\, (x^{k-1})'(x^{m-1}) d\,x$$

$$C_{km} = \int_{-0\cdot5}^{0\cdot5} \beta\, (x^{k-1})(x^{m-1})' d\,x$$

$$D_{km} = \int_{-0\cdot5}^{0\cdot5} \left[\beta\, (x^{k-1})(x^{m-1}) + (x^{k-1})'(x^{m-1})'\right] d\,x + \theta_1\, (-0\cdot5)^{k-1}(-0\cdot5)^{m-1} + \theta_2\, (0\cdot5)^{k-1}(0\cdot5)^{m-1}$$

$$E_{km} = \int_{-0\cdot5}^{0\cdot5} (x^{k-1})\,(x^{m-1})\,d\,x$$

$$F_{km} = \int_{-0\cdot5}^{0\cdot5} \mu\,(x^{k-1})\,(x^{m-1})\,d\,x \qquad k, m = 1, 2, ..., M \tag{22}$$

The eigenvalues (characteristic values) λ are found from the condition that the determinant of the system of equations given by Eq. (21) must vanish.

3. NUMERICAL RESULTS

In order to investigate the influence of stiffness of the supports on the free vibration characteristics of Timoshenko beams, the first three eigenvalues of Timoshenko beam with the first and the second type of translational and rotational springs (Fig. 1) are calculated for h/L=0.005 and three dimensional plots of Tables 2, 3, 4, 5, 6 and 7 are provided in Figs. 2, 3, 4, 5, 6 and 7 to illustate how the frequency parameters change with the spring constants. The stiffness parameters κ_i and θ_i are taken as having the same values at all the supports denoted by

$\kappa_1 = \kappa_2 = \kappa,\ \theta_1 = \theta_2 = \theta$ for the beam with the first type of the springs, and by $\kappa_1 = 1 \cdot 10^8,\ \kappa_2 = \kappa,\ \theta_1 = \theta,\ \theta_2 = 0$ for the beam with the second type of the springs.

It is possible to simulate infinite support stiffness by setting the translational or rotational stiffness coefficient equal to $1 \cdot 10^8$ at all the supports for comparing the obtained results with the existing results of the classically supported Timoshenko beams. Therefore, comparison study of the pinned-pinned ($\kappa_1 = \kappa_2 = 1 \cdot 10^8$, $\theta_1 = \theta_2 = 0$) and clamped-clamped ($\kappa_1 = \kappa_2 = 1 \cdot 10^8$, $\theta_1 = \theta_2 = 1 \cdot 10^8$) Timoshenko beam with the classical solutions based on the Euler-Bernoulli beam theory [1] and the results of the Pseudospectral method given in the Ref. [5] is carried out, and the results are given in Tables 1b and 1c. Also, by setting the translational and rotational stiffness coefficients equal to zero at all the supports, a completely free beam situation can be obtained. Moreover, the convergence is tested in Table 1a by taking the number of terms $M = 6, 8, 10, 12, 14, 16, 18$.

It is not necessary to give the values of E, G and A of the beam in the calculations. Relationship between E and G is as follows:

$$G = \frac{E}{2(1+\nu)} \tag{23}$$

The dimensionless parameter β is defined as follows:

$$\beta = \frac{6k_s}{(1+\nu)} \frac{L^2}{h^2} , \tag{24}$$

where ν is the Poisson's ratio. In all of the following calculations, the rectangular cross-sectional beams with shear correction factor $k_s = 5/6$ are considered and, the Poisson's ratio is taken $\nu = 0.3$.

Table 1a. The convergence study of the first six dimensionless frequency parameters λ_i of the pinned-pinned Timoshenko beam for h/L=0.01

M	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
6	3.14178	6.29432	11.4465	16.5662	-	-
8	3.14133	6.28110	9.49597	12.8103	-	-
10	3.14133	6.28106	9.41871	12.5577	16.2856	20.0176
12	3.14133	6.28106	9.41760	12.5494	15.7087	18.8955
14	3.14133	6.28106	9.41760	12.5493	15.6755	18.7960
16	3.14133	6.28106	9.41760	12.5493	15.6748	18.7925
18	3.14133	6.28105	9.41759	12.5493	15.6748	18.7924
PS [5]	3.14133	6.28106	9.41761	12.5494	15.6749	18.7926

It is shown that the convergence with respect to the number of the power series terms is excellent in the considered cases. As it is observed from the Table 1a, the frequency parameter decreases as the number of the power series terms increases: It means that the convergence to the exact value is from above. Namely, by increasing the number of the polynomial terms, the exact value can be approached from above. It should be remembered that energy methods always overestimate the fundamental frequency, so with more refined analyses, the exact value can be approached from above.

From here on, the number of the power series terms M is taken as 16 in all of the numerical investigations, namely the size of the determinant is 32×32 .

Table 1b. Comparison study of the first six dimensionless frequency parameters λ_i of the pinned-pinned Timoshenko beam

Methods	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
Classical Solution [1]	3.14159	6.28319	9.42478	12.5664	15.7080	18.8496
h/L=0.005						
Present	3.14152	6.28265	9.42297	12.5620	15.6996	18.8351
PS [5]	3.14153	6.28265	9.42298	12.5621	15.6997	18.8352
h/L=0.02						
Present	3.14053	6.27470	9.39630	12.4993	15.5783	18.6281
PS [5]	3.14053	6.27471	9.39632	12.4994	15.5784	18.6282
h/L=0.05						
Present	3.13499	6.23136	9.25536	12.1813	14.9926	17.6809
PS [5]	3.13498	6.23136	9.25537	12.1813	14.9926	17.6810

Table 1c. Comparison study of the first six dimensionless frequency parameters λ_i of the clamped-clamped Timoshenko beam.

Methods	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
Classical Solution [1]	4.73004	7.85320	10.9956	14.1372	17.2788	20.4204
h/L=0.005						
Present	4.72962	7.85161	10.9916	14.1292	17.2652	20.3965
PS [5]	4.72963	7.85163	10.9917	14.1294	17.2651	20.3985
h/L=0.02						
Present	4.72348	7.82816	10.9340	14.0153	17.0676	20.0845
PS [5]	4.72350	7.82817	10.9341	14.0154	17.0679	20.0868
h/L=0.05						
Present	4.68991	7.70350	10.6401	13.4610	16.1589	18.7360
PS [5]	4.68991	7.70352	10.6401	13.4611	16.1590	18.7318

Table 2. Variation of the first frequency parameter of the Timoshenko beam with the first type of translational and rotational spring parameters ($\kappa_1 = \kappa_2 = \kappa$, $\theta_1 = \theta_2 = \theta$) for h/L=0.005.

λ_1	θ								
κ	10^0	10^1	10^2	10^3	10^4	10^5	10^6	10^7	10^8
10^0	1.18562	1.18767	1.18829	1.18834	1.18839	1.18839	1.18839	1.18839	1.18839
10^1	2.05403	2.08824	2.09871	2.09999	2.10010	2.10013	2.10013	2.10013	2.10013
10^2	3.02962	3.36110	3.49767	3.51575	3.51761	3.51780	3.51782	3.51782	3.51782
10^3	3.35418	4.04250	4.45913	4.52532	4.53236	4.53306	4.53312	4.53314	4.53314
10^4	3.39417	4.14377	4.62208	4.70004	4.70834	4.70917	4.70926	4.70926	4.70926
10^5	3.39825	4.15427	4.63905	4.71821	4.72663	4.72749	4.72758	4.72758	4.72758
10^6	3.39865	4.15531	4.64074	4.72003	4.72848	4.72932	4.72941	4.72942	4.72942
10^7	3.39870	4.15542	4.64092	4.72021	4.72866	4.72951	4.72959	4.72960	4.72960
10^8	3.39870	4.15543	4.64094	4.72023	4.72867	4.72953	4.72962	4.72962	4.72962

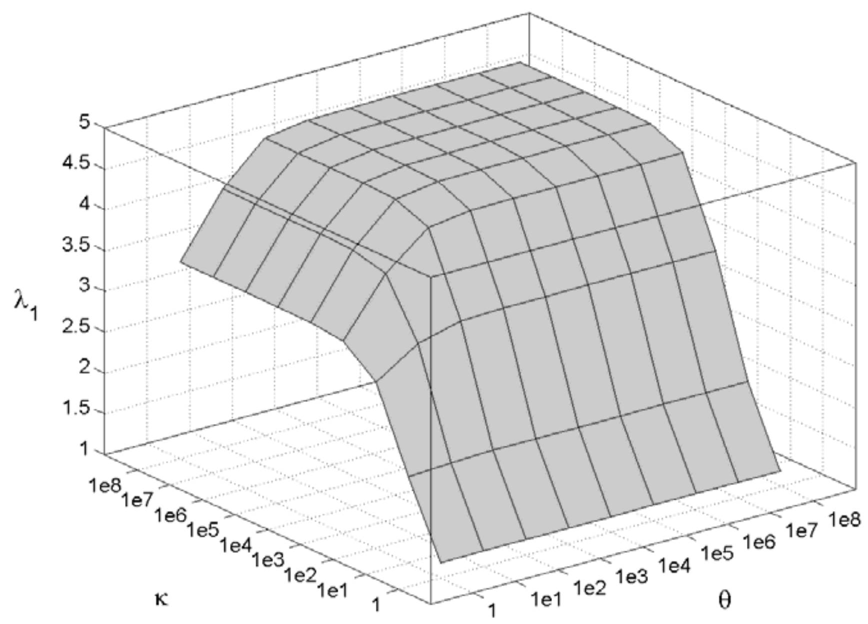

Figure 2 Plot of the first frequency parameter of the Timoshenko beam with the first type of translational and rotational spring parameters ($\kappa_1 = \kappa_2 = \kappa$, $\theta_1 = \theta_2 = \theta$) for h/L=0.005

Table 3. Variation of the second frequency parameter of the Timoshenko beam with first type of translational and rotational spring parameters ($\kappa_1 = \kappa_2 = \kappa$, $\theta_1 = \theta_2 = \theta$) for h/L=0.005

λ_2	θ								
κ	10^0	10^1	10^2	10^3	10^4	10^5	10^6	10^7	10^8
10^0	2.23329	2.93331	3.14411	3.17028	3.17298	3.17325	3.17326	3.17326	3.17326
10^1	2.93324	3.27082	3.40294	3.42020	3.42199	3.42216	3.42218	3.42218	3.42218
10^2	4.66386	4.66436	4.66463	4.66467	4.66467	4.66467	4.66467	4.66467	4.66467
10^3	6.13445	6.52658	6.85859	6.91869	6.92519	6.92585	6.92591	6.92592	6.92592
10^4	6.39693	7.01038	7.61856	7.73905	7.75228	7.75362	7.75375	7.75377	7.75377
10^5	6.42369	7.06163	7.69987	7.82646	7.84036	7.84176	7.84190	7.84192	7.84192
10^6	6.42637	7.06675	7.70795	7.83512	7.84909	7.85049	7.85064	7.85065	7.85066
10^7	6.42663	7.06727	7.70876	7.83599	7.84996	7.85137	7.85151	7.85153	7.85153
10^8	6.42666	7.06732	7.70883	7.83607	7.85004	7.85146	7.85160	7.85161	7.85161

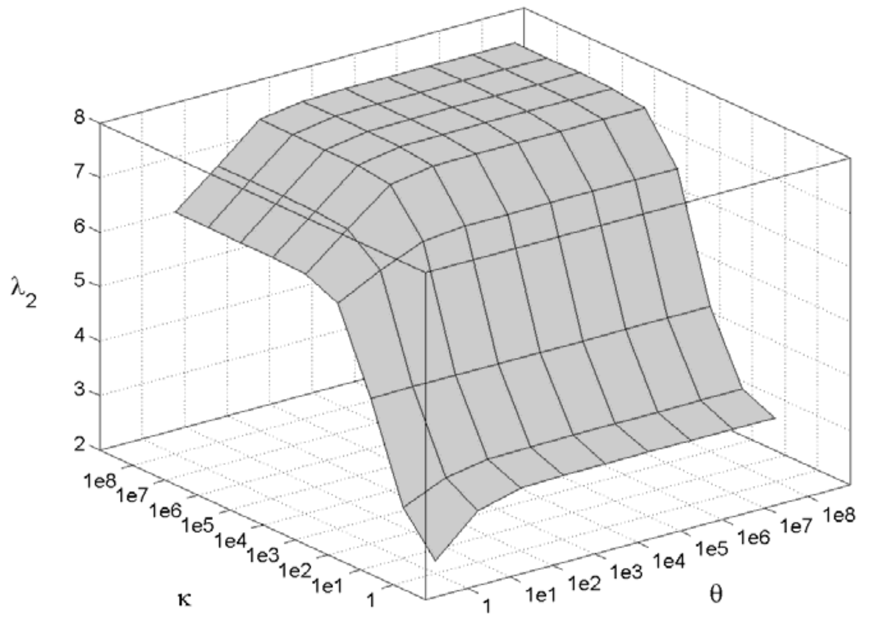

Figure 3 Plot of the second frequency parameter of the Timoshenko beam with the first type of translational and rotational spring parameters ($\kappa_1 = \kappa_2 = \kappa$, $\theta_1 = \theta_2 = \theta$) for h/L=0.005

Table 4. Variation of the third frequency parameter of the Timoshenko beam with the first type of translational and rotational spring parameters ($\kappa_1 = \kappa_2 = \kappa$, $\theta_1 = \theta_2 = \theta$) for h/L=0.005

λ_3	θ								
κ	10^0	10^1	10^2	10^3	10^4	10^5	10^6	10^7	10^8
10^0	5.06287	5.84564	6.22670	6.28046	6.28606	6.28662	6.28667	6.28668	6.28668
10^1	5.18944	5.90653	6.26595	6.31707	6.32240	6.32293	6.32299	6.32300	6.32300
10^2	6.16551	6.46210	6.64841	6.67711	6.68013	6.68043	6.68047	6.68047	6.68047
10^3	8.58315	8.66483	8.74316	8.75820	8.75985	8.76002	8.76004	8.76004	8.76004
10^4	9.42740	9.90465	10.5435	10.6942	10.7113	10.7130	10.7132	10.7132	10.7132
10^5	9.51316	10.0475	10.7730	10.9435	10.9628	10.9647	10.9649	10.9650	10.9650
10^6	9.52163	10.0616	10.7952	10.9674	10.9868	10.9888	10.9890	10.9890	10.9890
10^7	9.52248	10.0630	10.7974	10.9697	10.9892	10.9912	10.9914	10.9914	10.9914
10^8	9.52256	10.0632	10.7976	10.9700	10.9894	10.9914	10.9916	10.9916	10.9916

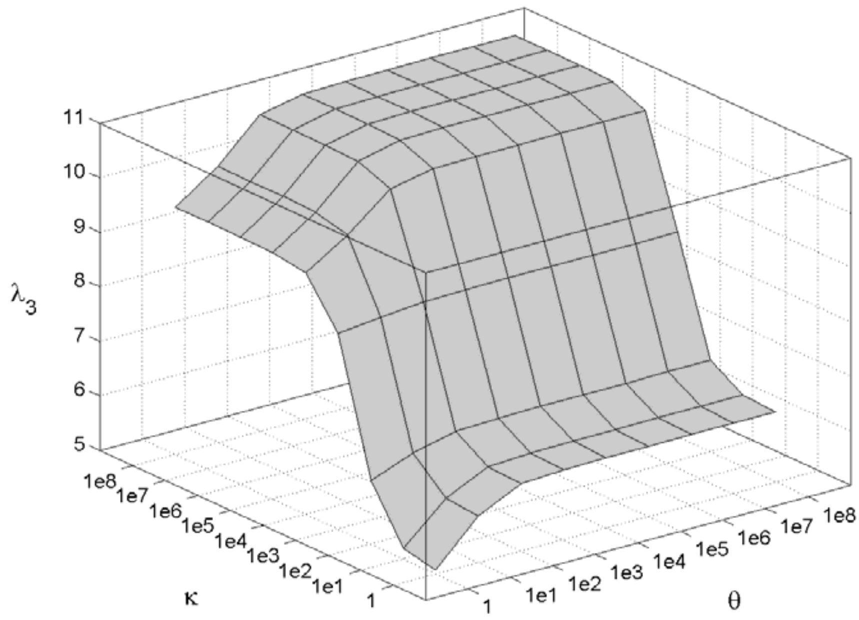

Figure 4 Plot of the third frequency parameter of the Timoshenko beam with the first type of translational and rotational spring parameters ($\kappa_1 = \kappa_2 = \kappa$, $\theta_1 = \theta_2 = \theta$) for h/L=0.005

Table 5. Variation of the first frequency parameter of the Timoshenko beam with the second type of translational and rotational spring parameters ($\kappa_1 = 1\cdot 10^8$, $\kappa_2 = \kappa$, $\theta_1 = \theta$, $\theta_2 = 0$) for h/L=0.005

λ_1	θ								
κ	10^0	10^1	10^2	10^3	10^4	10^5	10^6	10^7	10^8
10^0	1.53580	1.87927	1.99393	2.00832	2.00981	2.00996	2.00996	2.01000	2.01000
10^1	2.32645	2.53882	2.62612	2.63758	2.63876	2.63887	2.63890	2.63890	2.63890
10^2	3.10840	3.44112	3.61323	3.63759	3.64013	3.64039	3.64041	3.64041	3.64041
10^3	3.25657	3.64214	3.86128	3.89381	3.89723	3.89757	3.89760	3.89760	3.89760
10^4	3.27155	3.66227	3.88623	3.91964	3.92314	3.92350	3.92353	3.92354	3.92354
10^5	3.27304	3.66427	3.88871	3.92220	3.92572	3.92608	3.92611	3.92611	3.92611
10^6	3.27319	3.66447	3.88896	3.92247	3.92598	3.92633	3.92637	3.92637	3.92637
10^7	3.27321	3.66449	3.88900	3.92248	3.92600	3.92636	3.92639	3.92640	3.92640
10^8	3.27321	3.66451	3.88900	3.92250	3.92600	3.92636	3.92639	3.92640	3.92640

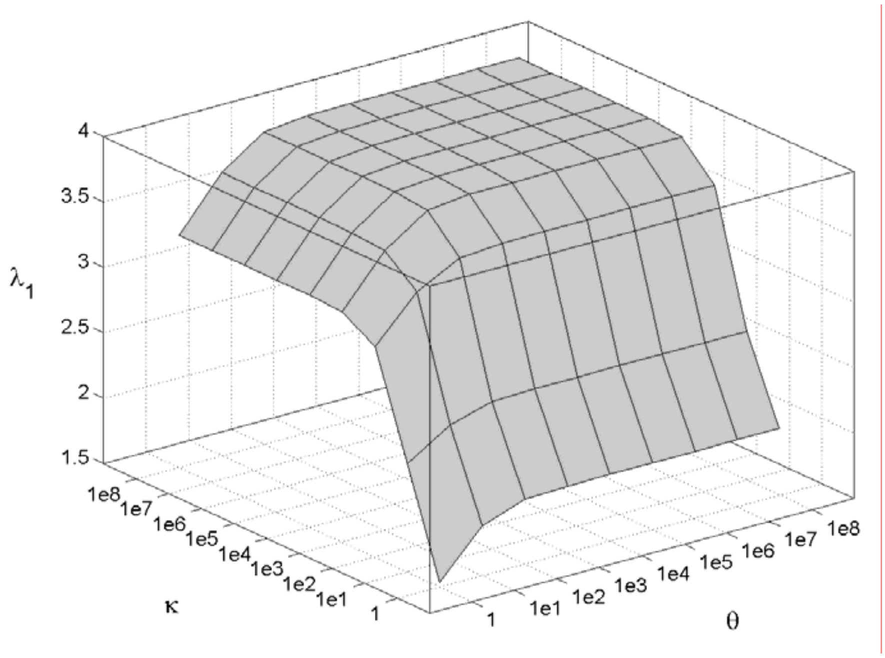

Figure 5. Plot of the first frequency parameter of the Timoshenko beam with the second type of translational and rotational spring parameters ($\kappa_1 = 1\cdot 10^8$, $\kappa_2 = \kappa$, $\theta_1 = \theta$, $\theta_2 = 0$) for h/L=0.005

Table 6. Variation of the second frequency parameter of the Timoshenko beam with second type of translational and rotational spring parameters ($\kappa_1 = 1\cdot10^8$, $\kappa_2 = \kappa$, $\theta_1 = \theta$, $\theta_2 = 0$) for h/L=0.005

λ_2	θ								
κ	10^0	10^1	10^2	10^3	10^4	10^5	10^6	10^7	10^8
10^0	4.04597	4.41059	4.65930	4.69882	4.70300	4.70341	4.70345	4.70346	4.70346
10^1	4.18444	4.51571	4.75112	4.78898	4.79299	4.79340	4.79344	4.79344	4.79344
10^2	5.19848	5.40970	5.58280	5.61218	5.61531	5.61563	5.61566	5.61566	5.61566
10^3	6.22083	6.53084	6.81736	6.86924	6.87483	6.87539	6.87544	6.87545	6.87545
10^4	6.34263	6.67206	6.98507	7.04279	7.04903	7.04966	7.04972	7.04973	7.04973
10^5	6.35415	6.68525	7.00058	7.05883	7.06513	7.06576	7.06583	7.06583	7.06583
10^6	6.35529	6.68656	7.00212	7.06042	7.06672	7.06735	7.06741	7.06742	7.06742
10^7	6.35541	6.68669	7.00227	7.06057	7.06688	7.06751	7.06757	7.06758	7.06758
10^8	6.35542	6.68671	7.00229	7.06059	7.06689	7.06753	7.06759	7.06760	7.06760

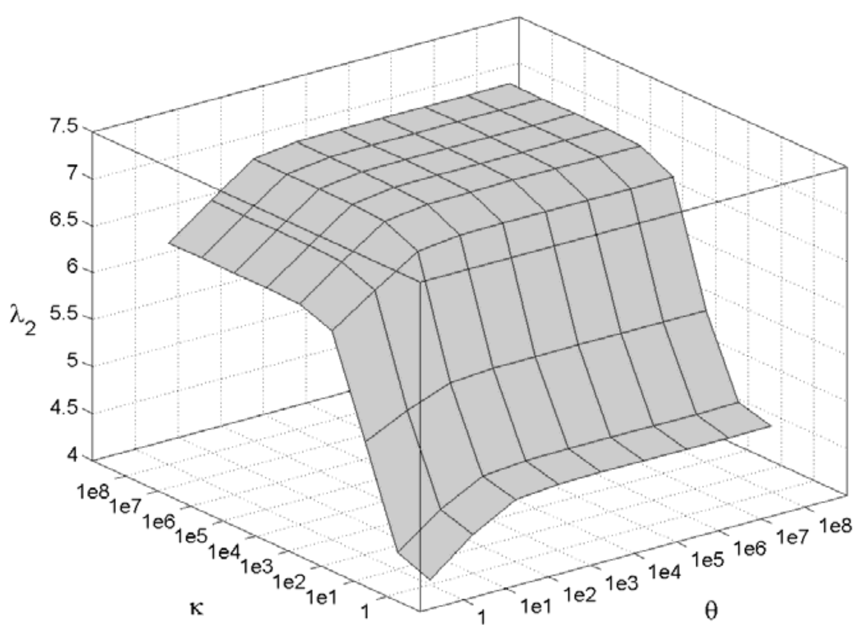

Figure 6. Plot of the second frequency parameter of the Timoshenko beam with the second type of translational and rotational spring parameters ($\kappa_1 = 1\cdot10^8$, $\kappa_2 = \kappa$, $\theta_1 = \theta$, $\theta_2 = 0$) for h/L=0.005

Table 7. Variation of the third frequency parameter of the Timoshenko beam with second type of translational and rotational spring parameter ($\kappa_1 = 1\cdot 10^8$, $\kappa_2 = \kappa$, $\theta_1 = \theta$, $\theta_2 = 0$) for h/L=0.005

λ_3	θ								
κ	10^0	10^1	10^2	10^3	10^4	10^5	10^6	10^7	10^8
10^0	7.13608	7.45243	7.78353	7.84774	7.85473	7.85544	7.85551	7.85551	7.85551
10^1	7.16108	7.47394	7.80273	7.86661	7.87356	7.87426	7.87433	7.87434	7.87434
10^2	7.44088	7.71432	8.01572	8.07553	8.08207	8.08272	8.08279	8.08279	8.08280
10^3	8.96347	9.19523	9.48258	9.54329	9.54998	9.55066	9.55072	9.55073	9.55073
10^4	9.42953	9.70263	10.0627	10.1424	10.1513	10.1522	10.1523	10.1523	10.1523
10^5	9.46878	9.74489	10.1107	10.1920	10.2010	10.2019	10.2020	10.2020	10.2020
10^6	9.47260	9.74899	10.1154	10.1968	10.2058	10.2067	10.2068	10.2068	10.2068
10^7	9.47298	9.74940	10.1158	10.1972	10.2063	10.2072	10.2073	10.2073	10.2073
10^8	9.47301	9.74944	10.1159	10.1973	10.2064	10.2073	10.2074	10.2074	10.2074

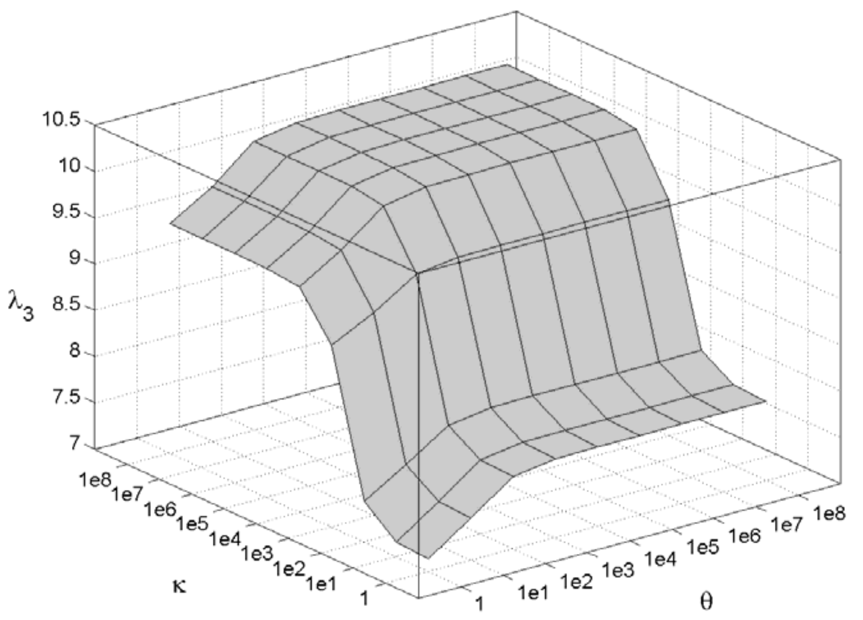

Figure 7. Plot of the third frequency parameter of the Timoshenko beam with the second type of translational and rotational spring parameters ($\kappa_1 = 1\cdot 10^8$, $\kappa_2 = \kappa$, $\theta_1 = \theta$, $\theta_2 = 0$) for h/L=0.005

It can be deduced that the results obtained from the present study are in good agreement with those of Lee and Schultz [5] as given in the Tables 1a-b-c. It should be remembered that, the eigenvalues obtained by using first order or higher order beam theories are lower than the corresponding eigenvalues obtained by the classical beam theory. It is observed from the Tables 1b-c that, the difference in the frequencies of the Euler-Bernoulli and Timoshenko beams becomes significant with increase of the mode numbers. For example, the value of the frequency of the pinned-pinned beam based on the classical theory is 3.14159 and of the pinned-pinned Timoshenko beam (for $h/L = 0.05$) 3.13499 for the first mode while they are 18.8496 and 17.6809 for the sixth mode. Also, as it is known and can be deduced from Table 1 that, with increase in the ratio of h/L, the dimensionless frequencies of the Timoshenko beams decreases compared with the frequencies of Euler-Bernoulli beams. However, the two solutions are very close to each other for small values of h/L (i.e. h/L=0.005) as seen from Tables 1a-b-c.

It is seen from the tables and the figures that, translational springs are much more effective on the frequency parameters than rotational springs. For example, in Table 3, when the spring parameter κ is taken constant value of 10^0 and the parameter θ is changed from 10^0 to 10^8, the frequency parameter λ_2 changes from 2.23329 to 3.17326 but, while the parameter θ is taken as 10^0 and the parameter κ is changed from 10^0 to 10^8, the frequency parameter λ_2 changes from 2.23329 to 6.42666.

Increment in the values of parameters κ and θ is more effective on the first frequency parameter of the beam than the second and third frequency parameters. For instance, for the beam with the first type of the springs, when the parameters κ and θ are both changed from $1 \cdot 10^0$ to 10^8, the first frequency parameter λ_1 changes from 1.18562 to 4.72962, namely, λ_1 becomes four times greater in this change. On the other hand, λ_2 and λ_3 increase approximately 3.5 and 2.17 times, respectively in the considered change.

When the values of κ and θ are greater than $\kappa = 10^5$ and $\theta = 10^5$, then, there is no remarkable change in the frequeny parameters. This situation can be observed from the flat area of the Figs. 2-7. Also, it is evident from the obtained values of frequency parameters that, when the parameters κ and θ are taken as $\kappa = \theta = 1 \cdot 10^8$, then, the beam can be considered as a beam fixed at the both ends.

4. CONCLUSIONS

The free vibration of elastically supported Timoshenko beams have been investigated for different support stiffnesses. To compare the obtained results with the previously published results, the frequency parameters of pinned-pinned and clamped-clamped Timoshenko beams, which are special cases of the present problem are calculated. Using the Lagrange equations with the trial functions in the power series form and satisfying the constraint conditions by the use of very stiff springs is a very good way for studying the free vibration characteristics of the elastically supported beams. Numerical calculations have been carried out to clarify the effects of support stiffnesses on the free vibration characteristics of the considered beams. It is observed from the investigations that, all of the obtained results are very accurate and may be useful to other researchers so as to compare their results.

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