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DESIGN AND RELIABILITY BASED OPTIMIZATION OF A 2D ARCH BRIDGE

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ABSTRACT

The aim of this work is to design and optimize a 2D Arch Bridge with reliability based optimization concept that is commonly used in transportation. After the arch bridge is modeled according to its original layout, the formulation of reliability-based optimization and other optimization techniques used in the work are presented. Then the bridge is optimized using various optimization methods under the allowable stress failure probability and different values of variation coefficient of random variables. Finally shape and size optimization of 2D arch bridge is performed. It is concluded that the reliability based optimization concept offers an optimum design balancing both the weight and the safety better than the deterministic ones.
Keywords: Reliability, Reliability index, structural optimization, reliability based design optimization, failure probability.

İKİ BOYUTLU KÖPRÜ KİRİŞİNİN TASARIMI VE GÜVENİLİRLİĞİNE DAYALI OPTİMİZASYONU

ÖZET

Bu çalışmanın amacı ulaştırmada sıklıkla kullanılan iki boyutlu köprü kirişinin tasarımını ve güvenilirliğe dayalı optimizasyonunu yapmaktır. Köprü kirişi orijinaline uygun olarak modellendikten sonra, güvenilirliğe dayalı optimizasyonun formülasyonu ve kullanılacak olan bazı optimizasyon metodları sunulmaktadır. Daha sonra kiriş izin verilen gerilme göçme olasılığı ve rasgele değişkenlerin farklı değerdeki varyans katsayıları altında optimizasyon metodları ile optimize edilmektedir. Son olarak kirişin şekil ve boyut optimizasyonu gerçekleştirilmektedir. Güvenilirliğe dayalı optimizasyonun, deterministik olana nazaran hem güvenlik hem de ağırlık olarak daha dengeli bir tasarım sunduğu söylenebilir.
Anahtar Sözcükler: Güvenilirlik, güvenilirlik indeksi, yapısal optimizasyon, güvenilirliğe dayalı optimization, göçme olasılığı.

1. INTRODUCTION

The uncertainties of the structural parameters and the scatter from their nominal values are inherent and unavoidable due to the fabrication, workmanships, human and so on in most engineering applications. These uncertainties play very important role in the failure of structures.

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To assess this influence, stochastic analysis methods and the development of it that has taken place during the last two decades [1] has stimulated interest [2].

Reliability analysis leads to safety measures that a design engineer has to take into account due to the aforementioned uncertainties. So the definition of the some basic design variables is required for uncertainty in different forms, material properties and loads, in addition to boundary conditions, methods of modeling and analysis, failures in the reliability analyses. Probability theory is used to measure the ability of structures to fulfill its design purpose for some time period. This ability is defined as the reliability of an engineering system viewed as the probability of its satisfactory performance. In estimating this probability, system uncertainties are modeled as random variables with mean values, variances, and probability distribution functions [3].

For structural reliability assessment purposes many methods have been used. These methods are based on simulation techniques and approximation (moment) methods. As Monte Carlo (MC), Latin Hyper Cube (LHC), Importance Sampling Point (IS) [4, 5] are well known simulation techniques, First Order Reliability Method (FORM) and Second Order Reliability Method (SORM) [6-8] are well known approximation methods.

Structural optimization has recently undergone substantial progress. The techniques currently available have matured to the point that optimization methods are being added to many existing commercial finite element codes. However, most of these developments deal only with deterministic parameters. For the rational design it is crucial to account for uncertain properties of material, loading and geometry as well as the mathematical model of the system. Moreover, reliability performances should be introduced as the most rational safety measures. Deterministic optimization enhanced by reliability performances and formulated within the probabilistic framework is called reliability-based design optimization (RBDO) [9]. Compared to the basic deterministic-based optimization problem, a RBDO problem considers additional non-deterministic constraint functions.

This paper presents a procedure to perform the reliability-based optimum design of 2D arch bridges. After the arch bridge is modeled according to its original layout, Finite Element (FE) discrezetion and RBDO formulation of 2D arch bridge are presented. Size optimization of the bridge is performed under the allowable stress failure probability constraint and different values of variation coefficient of random variables. Finally, size and shape optimization of the bridge based on RBDO concept are performed.

2. DESCRIPTION AND THE PROPERTIES OF THE 2D ARCH BRIDGE

Before a truss bridge can be constructed, important specifications about its shape, dimensions, member forces, strain, and stress, are meticulously calculated by design engineers that allow the bridge to maintain its structural integrity. Fig. 1 illustrates an existing arched steel bridge in Hiltrup, Münster,Germany. In this work RBDO of this bridge are performed. However the bridge is examined in 2D. The arch bridge consists of three parts as the upper chord, the rod and the lower chord. The bridge is formed of 33 nodes and 43 elements. The upper and lower chord of elements have hollow rectangular and I shape cross sections respectively while the rod elements have circular cross sections. Also, as the upper and lower chords of elements type are Beam2D, the rod elements type is Truss2D. The shape of the 2D arch Bridge is given in Fig. 2.

There is one fixed support in the horizontal and vertical direction at the bottom left node, and one fixed support in the vertical direction only at the bottom right node. The total span of the bridge is 87,30 meters. The definition of 2D Arch Bridge is performed by Finite Element Methodology. Fig. 3a, b and c show this definition according to FEM.

Figure 1. Arch steel bridge in Münster-Hiltrup

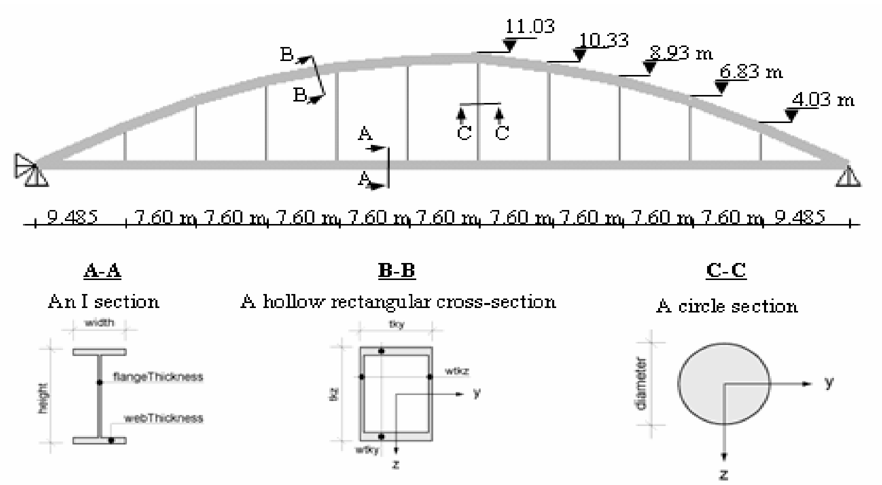


Figure 2. The topology and geometrical properties of the cross-section of 2D Arch Bridge

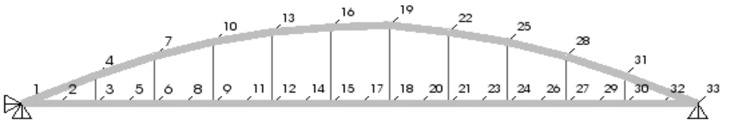
The design load for the bridge calculated according to AASHTO standard is given below. As known, these loads symbolize the truck load. And, since trucks travel directly on the superstructure, all part of bridge is subjected to vibration and must be designed under the impact load. AASHTO prescribes empirically that the impact factor expressed as a portion of live load. It is defines as follows [10]:

$$I=50/(L+125) \leq 0.30 \qquad (1)$$

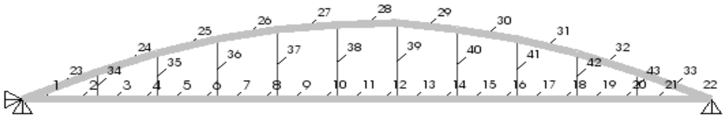
Where, L is expressed in feet and it symbolizes the span of the bridge. The structural system is analyzed after the load vector of the system is multiplied by the value calculated adding the impact factor to one. Therefore, the design load multiplied by this value is 423.92 kN.

2.1. Dimensions Of Element Sections

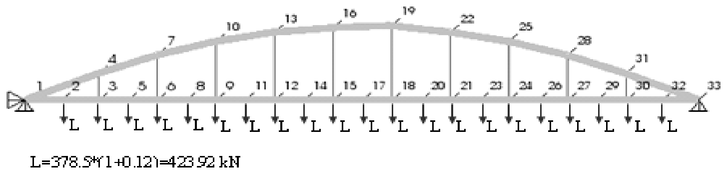
As mentioned before, element cross sections to form the bridge are different from each other. Dimensions of the cross sections are taken from original project of steel arch bridge and those are presented in Fig. 4a, b, and c. The same figure also shows the design variables adapted as dimensions of the cross sections in the current work for the size optimization.



a. Numbering of nodes



b. Numbering of elements



c. Load case for 2D Arch Bridge

Figure 3. The definition of 2D Arch Bridge

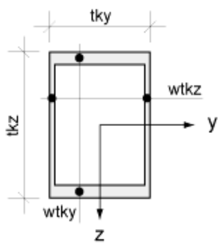
3. Reliability Based Optimization

Bridges can be designed more elegant and economical through the development of computer technologies and optimum design approaches. The self-weight of the structure increases quickly as its span expands. That is why it is very important to design the structures with a possible minimum self-weight satisfying certain design requirements.

In any optimum design problem, some certain criteria must be established to evaluate a reasonable solution. For a structure, typical criteria may be (1) minimum cost; (2) minimum weight; (3) minimum construction time; (4) minimum workmanship; (5) minimum cost of owner's products [10]. The criterion of minimum weight is generally used in optimum design.

Conventional optimization, which is generally used in practice, is a deterministic process. It has been observed that the optimum structures obtained through deterministic optimization do not necessarily have high reliability. The optimization based on reliability concept will lead to more consistent safety in the structural system.

Two main type of reliability-based formulations, the component and system reliability index-based optimization, have been developed. However, there are also some different optimum design problems available in these main categories, i.e. determination of optimum values of design variables to minimize the structural cost or weight subject to the specified allowable failure probability of element or structure. Another design problem is determination of design variables to minimize the expected total cost which is composed of failure cost, which is a function of the probability of failure, and structural cost.

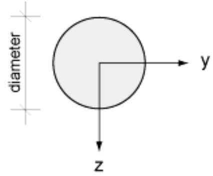


Type of elements: **Beam2D**
Cross section type: **Hollow Rectangular**

The elements forming the upper chord: **23… 33**

The size of tky: **1150 mm** The size of wtky: **30 mm**
The size of tkz: **800 mm** The size of wtkz: **22 mm**

a**.** Cross section of the elements forming the upper chord

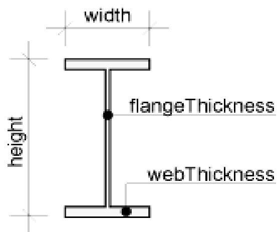


Type of elements: **Truss2D**
Cross section type: **circle**

The elements forming the rod: **34… 43**

The size of diameter: **110 mm**

b. Cross section of the elements forming the rod



Type of elements: **Beam2D**
Cross section type: **I shape**

The elements forming the lower chord: **1… 22**

The size of width**: 1150 mm** The size of web Thickness: **30 mm**
The size of height: **800 mm** The size of flange Thickness: **22 mm**

c. Cross section of the elements forming the lower chord

Figure 4. Properties of the cross sections used in the steel arch bridge

The optimum design problems to minimize the structural weight under the constraints on the probabilities of failure of the members are adopted in this work. The allowable tensile stress and loads together are assumed to have stochastic nature. The design variables are the geometrical properties of the cross-sections. The failure criteria of the members are expressed as a function of the strength of the member and the applied loads. The formulation of the optimization problem outlined above as follows:

$$\min_{A}\left\{ W=\sum_{i=1}^{m}\rho l_i A_i \quad | \; P_{fi} < P_{fa} \right\} \tag{2}$$

Where W is the weight of structural system, l_i, A_i are the length and cross-section area of member i respectively, P_{fi} is the failure probability of member i, P_{fa} is the specified allowable probability of failure, and m is the number of members of structural system. The safety margin or limit state function of member i $g(A_i \; i=1,\ldots,m)$ is calculated as:

$$g_i(A) = R_{ai} A_i - \sum_{j=1}^{\ell} b_{ij}(A) L_j \tag{3}$$

Where R_{ai} is the allowable stress of member i, $b_{ij}(A)$ is the load coefficient of member i with respect to L_j, L_j is the applied load, and ℓ is the number of the applied loads.

The probability of failure of member *i* is calculated by using First Order Second Moment (FOSM) method in this work.

$$P_{fi} = P(g_i(A) \le 0) = \Phi(-\beta) \tag{4}$$

Also it is possible to formulate the reliability of member *i* depending on P_{fi} which is given by 1-P_{fi}. It can be seen from Eq. (3) that the strength of the members (R_{ai} A_i) is easily determined by specifying the material and dimension of the member. Evaluation of the internal forces of the member is very complex and it is derived by applying Matrix Method (see more details [11-12]).

It is assumed that while the allowable stress of the members and the applied loads are statistically independent Gaussian random variables, the cross-section areas and lengths of members are deterministic as mentioned before. The mean value of the allowable stress and loads are taken as 212 N/mm^2 and 423.92 kN respectively. Failure of the members is assumed to occur due to tension or compression. Here, the allowable tension stress and compression stress are taken as the same. On the other hand in the design process, the values of the coefficients of variation for loads and stresses are specified as 0.20, 0.05 respectively. The young modulus, E, is taken as 21x10^{+4} N/mm^2. The sum of the probabilities of failure of the members(P_{fa})is specified to be 10^{-5}, and the probabilities of failure are equally allocated to all members, i.e.

P_{fai}=P_{fa}/43≈ 2.27x10^{-6} (i=1, 2,…,43)

The optimum design problem is considered to minimize the structural weight subject to the constraint on the structural elements probabilities of failure. It is also possible to define the constraint on the structural probability of failure as follows;

$$\sum_{i=1}^{43} P_{fai}(A_i) \le P_{fa} \approx 10^{-5}$$

Where, $Pf_{ai}(A_i)$ shows the value of failure probability of element and it is a function of the cross-sectional areas of the elements. The allowable probability of the member taken into account in the design process is 2.27x10^{-6}. A wide variety of optimization algorithms based on deterministic or stochastic techniques have been developed. However, there is no unique optimization algorithm which gives the best and reliable result in almost engineering application. So, for practical structural optimization it is necessary to have a set of optimization methods at hand and to be able to switch easily between them [13]. In this work, an optimization component created in Institute for Computational Engineering, Faculty of Civil Engineering, Ruhr University Bochum, Bochum, Germany was used and it is combined with a Finite element program called **miniFE** which is also available in the institute. The optimization component has the optimization methods and packages [13] which are listed in Table 1.

Table 1. Provided optimization algorithms and packages

Name	Method(s)	Author	Impl. language
DONLP	SQP	Spellucci	Fortran
DOT	FR/BFGS/SLP/MMFD/SQP	Vanderplaats Inc.	Fortran
EVOL	Evolutionary strategy	Grill	C++
FSQP	Feasible SQP	Lawrence et al.	C
NLPQL	SQP	Schittkowski	Fortran
SCP	CONLIN/MMA/SCP	Zillober	Fortran

To calculate the response of bridge under the random influence, a method is added to **miniFE**. The method is responsible of applying Matrix Method summarized above. At the beginning, the optimizer gets the data for the actual optimization problem and an appropriate optimization method is selected. Inside the optimization loop mainly two things happen: First, the problem functions and, if needed, their derivatives are evaluated. In the case that derivatives are needed but can not be computed analytically, they are numerically approximated by finite differences. Next, the optimization algorithm, according to its internal logic, generates the next, improved design vector based upon the previous function evaluations. These two steps are repeated until some termination criterion is met [13]. In the meantime, all FE calculation and operation of checking the constraint is fulfilled in **miniFE** depending on the data taken from the optimization component. And **miniFE** send some data to the optimization component such as violation value of constraints in order to recover the design vector.

4. RELIABILITY BASED DESIGN OPTIMIZATION OF THE 2D ARCH BRIDGE

The system is optimized by various optimization methods. The results are given in Table 2. It can be easily seen from Table 2 that the results obtained by EVOL are smaller than the results obtained by other methods. Therefore, EVOL method will be used only for the following comparisons. The variation in the value of design variables, objective function and violation of the constraint taken as the failure probabilities of the members in the EVOL design process are shown in Fig. 5.

To illustrate the effect of the value changes in the coefficient of variations of the random variables on the weight of the 2D Arch Bridge, it is optimized with various combination of the coefficient of variations, as shown in Table 3. As the coefficient of variation of the load (CV_L) becomes large, the weight of the bridge slightly increases. However, the weight of the bridge more increases when the coefficient of variation of the allowable stress becomes large. Therefore, it can be noted from Table 3 that the optimum design obtained for the design problem are sensitive to the changes in the coefficient of variations.

Table 2. The results for the 2D Arch Bridge

Design variables	EVOL (mm)	SQP	SCP	DONLP	Initial values of design variables
width	1259,87	1454,38	1015,77	958,78	950 (mm)
height	622,003	1499,76	762,97	575,85	550
flange Tickness	28,56	15,03	26,4	33,185	20
web Tickness	21,54	11,41	22,34	26,36	15
tky	1195,94	656,49	1164,62	952,12	950
tkz	497,88	447,26	730,91	550,38	550
wtkz	20,76	27,25	20,12	19,22	20
wtky	18,85	39,35	19,61	30,007	15
diameter	98,8	107,19	148,55	221,89	85
Mass (kN)	948,43	955,35	1064,22	1154,76	

Figure 5. Visualization of EVOL design process

Table 3. Optimization results for the various coefficient of variation

(CV_R,CV_L) Design variables	(0.03,0.10)	(0.10,0.10)	(0.05,0.20)	(0.03,0.40)	(0.10,0.40)	Initial values of design variables
width	702.2249	1487.6048	1260.0767	840.7746	1574.0659	750 (mm)
height	1124.2932	698.5644	873.4749	758.5796	1053.9875	450
flange Tickness	16.7018	34.2694	24.0164	31.0509	29.2875	18
web Tickness	23.8906	24.1755	16.0654	24.1799	17.1926	12
tky	1067.7212	1408.7629	1279.2606	1132.5518	1446.2451	850
tkz	444.2323	689.3006	440.2903	464.1296	494.2169	350
wtkz	20.6124	25.072	21.1143	24.8231	29.2369	20
wtky	17.8016	14.7911	16.2317	18.6691	14.1786	15
diameter	89.4374	111.9584	99.336	107.0773	119.6355	105
Mass (kN)	824.55	1266.18	948.025	1021.66	1376.02	
$\sum P_{fi}$ (i=1,…,43)	5.58E-05	6.14E-05	5.37E-05	4.97E-05	5.67E-05	

Especially the changes in the variation coefficient of the allowable stress (CV_R) effect the value of the objective function much. This is clearly shown in Fig. 6 and Table 4. As mentioned before EVOL method is used in the design process only. This is because EVOL method is not sensitive to the change in the initial value of the cross-section. This conclusion can easily be seen in both the cases where the initial value of the cross-section area is taken as given in Table 2 and Table 3, the EVOL method finds the same mass.

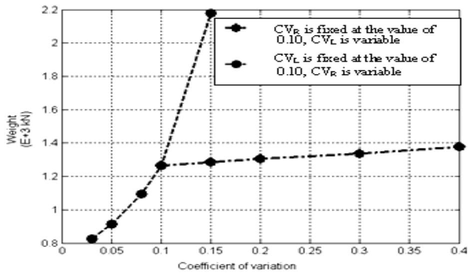


Figure 6. Graphical representation of the coefficient of variation versus the weight of the bridge

Table 4. Details of the optimization results for the various coefficient of variation of the random variables

CV_R, CV_L	0.10, 0.10	0.10, 0.15	0.10, 0.20	0.10, 0.30	0.10, 0.40	Symbol in the above figure
Mass (e+3 kN)	1,266	1,285	1,305	1,336	1,376	
CV_R, CV_L	0.03, 0.10	0.05, 0.10	0.08, 0.10	0.10, 0.10	0.15, 0.10	
Mass (e+3 kN)	0,824	0,913	1,093	1,266	2,177	

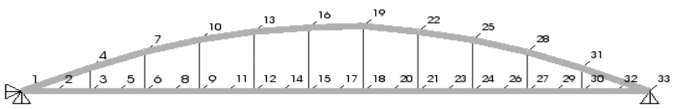
4.1. Size and Shape Optimization of 2D Steel Arch Bridge Based on RBDO Concept

The reliability based size and shape optimization of the bridge is evaluated to treat design variables specified for both cross-section area and the coordinates of the joints simultaneously, to obtain the minimum weight of the bridge under the probability of failure stress. In addition to the design variables which are the cross-sectional properties of element sections and are totally 9 distinct design variables, y coordinates of the upper chord joints are also taken as coordinates design variables. Thus, the problem has 9 cross-sectional and 5 coordinate design variables, a total of 14 independent variables. Also, due to the practical view and the symmetry the problem has five dependent coordinate variables for the upper chord. In other words, y coordinates of the five upper chord joints, 19-22-25-28-31, are linked to the y coordinates of the other upper chord joints, 4-7-10-13-16, respectively. For this case, the optimum results are presented in Table 5. The variations of the design variables, objective function, constraints and the optimum shape of the space truss are shown in Fig. 7.

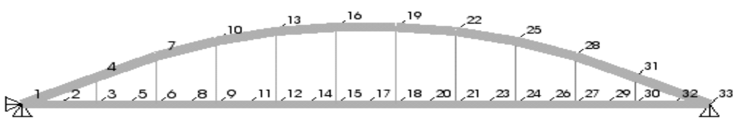
Table 5. The optimum results after the shape optimization

Design variables	Optimum values of design variables	Initial values of design variables
width	959,2145	750 (mm)
height	1222,2084	450
flange Tickness	27,9357	18
web Tickness	9,1676	12
tky	1193,332	850
tkz	674,3453	350
wtkz	26,8426	20
wtky	17,3294	15
diameter	109,9581	105
y coordinates of the nodes 4 and 31	4559,2929	4030
y coordinates of the nodes 7 and 28	8117,3044	6830
y coordinates of the nodes 10 and 25	10665,2471	8930
y coordinates of the nodes 13 and 22	12309,9603	10330
y coordinates of the nodes 16 and 19	13013,2777	11030
Mass (kN)	1217,6	
$\sum P_{fi}$ (i=1,…,43)	6,87E-05	

The values of the coefficient of variations of the random variables (CV_R, CV_L) are taken as 0.10 and 0.40 respectively in the EVOL design process. Comparing the sixth column of Table 3 with the second column of Table 5 it is stated that the weight reduction, which is % 11.52, was obtained in Table 5 with respect to the optimization listed in Table 3. The optimum value of the coordinates of the design variables are also illustrated in Table 5.



Initial shape (CV_R, CV_L) = 0.10, 0.40



Optimum shape (CV_R, CV_L) = 0.10, 0.40

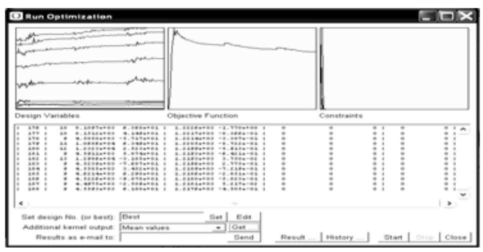


Figure 7. Optimum shape of the 2D arch bridge after the RBDO and visualization of EVOL design process

4. OBSERVATIONS AND CONCLUSION

Through the optimization of the typical 2D Arch Bridge under the probabilistic constraints; the following conclusions are drawn;

It is concluded that the algorithms coded for RBDO in the study are accurate and efficient. The change in the coefficient of variation affects the weight of the structure. The change in the coefficient of variation of the allowable stress is more active than the change in the coefficient of the variation of the load. The algorithm also enables of performing reliability-based configuration optimization. The shape optimization process provides significant reduction in the weight of the structure. The design obtained by RBDO balances both the cost and the safety better than the deterministic approach.

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