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STATIC ANALYSIS OF SHEAR DEFORMABLE RECTANGULAR PLATES ON WINKLER-PASTERNAK FOUNDATION

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ABSTRACT

Static analysis of shear deformable plates resting on two-parameter foundations is presented by the method of discrete singular convolution (DSC). The influence of foundation parameters on the deflections of the plate has been investigated. Numerical studies are performed and the DSC results are compared well with other analytical solutions and some numerical results.
Keywords: Thick plates, discrete singular convolution, elastic foundation.

WINKLER-PASTERNAK ZEMİNE OTURAN KAYMA DEFORMASYONLU DİKDÖRTGEN PLAKLARIN STATİK HESABI

ÖZET

İki parametreli zemine oturan kayma deformasyonlu plakların statik analizi için ayrık tekil konvolüsyon yöntemi sunulmuştur. Zemin parametrelerinin deformasyon üzerine etkisi incelenmiştir. Sayısal uygulama yapılmış ve elde edilen sonuçlar diğer analitik ve bazı sayısal çözüm yöntemlerinin verdiği sonuçlar ile karşılaştırılmıştır.
Anahtar Sözcükler: Kalın plak, ayrık tekil konvolüsyon, elastik zemin.

1. INTRODUCTION

Plates on elastic foundation have wide applications in pressure vessels technology such as petrochemical, marine and aerospace industry, civil, and mechanical engineering. Long list of references on dynamic and bending analysis of thin and thick plates on elastic foundation are given, for example, in References [1,2,10,11]. Some selected works in this research topic includes those of Liew et al. [3], Teo and Liew [4], Wang et al. [5], Kobayashi and Sonoda [6], Civalek [7,9], Daloğlu et al.[12], Ayvaz et al.[13], Omurtag and Kadıoğlu [14] and Kadıoğlu and Omurtag [15].

2. DISCRETE SINGULAR CONVOLUTION (DSC)

For brevity, consider a distribution, T and $\eta(t)$ as an element of the space of the test function. A singular convolution can be defined by [17].

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$$F(t) = (T * \eta)(t) = \int_{-\infty}^{\infty} T(t-x)\eta(x)dx \tag{1}$$

where $T(t-x)$ is a singular kernel. The DSC algorithm can be realized by using many approximation kernels. However, it was shown [18-21] that for many problems, the use of the regularized Shannon kernel (RSK) is very efficient. The RSK is given by [22]

$$\delta_{\Delta,\sigma}(x-x_k) = \frac{\sin[(\pi/\Delta)(x-x_k)]}{(\pi/\Delta)(x-x_k)} \exp\left[-\frac{(x-x_k)^2}{2\sigma^2}\right]; \ \sigma>0 \tag{2}$$

where $\Delta=\pi/(N\text{-}1)$ is the grid spacing and N is the number of grid points. In the DSC method, the function f (x) and its derivatives with respect to the x coordinate at a grid point x_i are approximated by a linear sum of discrete values $f(x_k)$ in a narrow bandwidth $[x\text{-}x_M, x\text{+}x_M]$. This can be expressed as [23]

$$\left.\frac{d^n f(x)}{dx^n}\right| = f^{(n)}(x) \approx \sum_{k=-M}^{M} \delta^{(n)}_{\Delta,\sigma}(x_i - x_k) f(x_k)\,; \ (n=0,1,2,..., \tag{4}$$

where superscript n denotes the nth-order derivative with respect to x. Detailed formulations for these differentiation coefficients can be found in references [24-26].

3. FUNDAMENTAL FORMULATIONS

The governing equations for bending of Mindlin plates on two-parameter elastic foundation can be given in the following dimensionless form as [3]

$$\frac{\partial^2 \Psi_X}{\partial X^2} + \beta^2 \frac{1-\upsilon}{2}\frac{\partial^2 \Psi_X}{\partial Y^2} + \frac{1+\upsilon}{2}\beta\frac{\partial^2 \Psi_Y}{\partial X \partial Y} + \alpha\chi\frac{\partial W}{\partial X} - \alpha\Psi_X = 0, \tag{5}$$

$$\frac{(1-\upsilon)}{2}\frac{\partial^2 \Psi_Y}{\partial X^2} + \beta^2 \frac{\partial^2 \Psi_Y}{\partial Y^2} + \frac{(1+\upsilon)}{2}\beta\frac{\partial^2 \Psi_X}{\partial X \partial Y} + \alpha\chi\frac{\partial W}{\partial Y} - \alpha\Psi_Y = 0, \tag{6}$$

$$(1+\frac{G_f}{\alpha})\left[\chi\frac{\partial^2 W}{\partial X^2} + \gamma\beta\frac{\partial^2 W}{\partial Y^2}\right] - \left[\frac{\partial \Psi_X}{\partial X} + \beta\frac{\partial \Psi_Y}{\partial Y}\right] + Q - \frac{K_F}{\alpha}W = 0. \tag{7}$$

In the equations given above, following new parameters are used.

$$X = x/a,\ Y = y/b,\ W = w/h,\ \chi = h/a,\ \beta = a/b,\ \gamma = h/b,\ \Psi_X = \psi_x$$
$$\alpha = 6\kappa(1-\upsilon)/\chi^2,\ \Psi_Y = \psi_y,\ Q = q/\kappa G\chi,\ K = a^4 k_f/D,\ G_f = a^2 G_F/D. \tag{8}$$

The moment resultants can be given by [4]

$$M_x = -D\left(\frac{\partial\psi_x}{\partial x} + \upsilon\frac{\partial\psi_y}{\partial y}\right), \quad M_y = -D\left(\upsilon\frac{\partial\psi_x}{\partial x} + \frac{\partial\psi_y}{\partial y}\right), \tag{9,10}$$

$$M_{xy} = -D\frac{1-\upsilon}{2}\left(\frac{\partial\psi_x}{\partial y} + \frac{\partial\psi_y}{\partial x}\right), \tag{11}$$

According to the DSC method, the normalized governing equations (Eqs.5-7) can be discretized into the following form

$$\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta)\Psi^X_{kj} + \beta^2\left(\frac{1-\upsilon}{2}\right)\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta)\Psi^X_{ik} + \beta\left(\frac{1+\upsilon}{2}\right)\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)\Psi^Y_{kj}\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)\Psi^Y_{ik}$$

$$\alpha\chi\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)W_{kj} - \alpha\Psi^X_j = 0 \tag{12}$$

$$\left(\frac{1-\upsilon}{2}\right)\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta)\Psi^Y_{kj} + \beta^2\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta)\Psi^Y_{ik} + \beta\left(\frac{1+\upsilon}{2}\right)\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)\Psi^Y_{kj}\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)\Psi^X_{ik}$$

$$\alpha\chi\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)W_{ik} - \alpha\Psi^Y_i = 0 \tag{13}$$

$$\left(1+\frac{G_f}{\alpha}\right)\left[\chi\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta)W_{kj} + \gamma\beta\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta)W_{ik}\right]$$

$$-\left[\chi\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)\Psi^X_{kj} + \beta\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta)\Psi^Y_{ik}\right] - \frac{K_F}{\alpha}W_j = -Q \tag{14}$$

Two types of boundary conditions, i.e., simply supported (S) and clamped (C) are taken into consideration. The normalized form of these boundary conditions can be given below:

$$W = 0, \qquad \Psi_X = 0, \qquad \text{and} \quad \Psi_Y = 0. \tag{15}$$

- Simply supported (S) edge at X=0, 1

$$W = 0,\ \frac{\partial\Psi_X}{\partial X} + \beta\upsilon\frac{\partial\Psi_Y}{\partial Y} = 0, \text{ and} \qquad \Psi_Y = 0. \tag{16}$$

- Simply supported (S) edge at Y=0, 1

$$W = 0,\ \upsilon\frac{\partial\Psi_X}{\partial X} + \beta\frac{\partial\Psi_Y}{\partial Y} = 0, \text{ and} \quad \Psi_X = 0. \tag{17}$$

4. NUMERICAL EXAMPLES

In the first test example, the central deflection of SSSS square plate under uniformly distributed load (h/a=0.05) is taken into account. Results from Buczkowski and Torbacki [8] and Timoshenko and Woinowsky-Krieger [16] are used to check present formulation for the deflection of plates on elastic foundation. These results are listed in Table 1. It is seen that the present method yields accurate results. Table 2 shows the comparison of the central deflection and bending moments of SSSS square plate under uniformly distributed load, with the solutions of Wang et al. [5] for different G_F.

Table 1. Comparison of the deflection $W/ (qa^4 /100D)$ of SSSS square plate under uniformly distributed load (h/a=0.05)

K	Analytical Solution Ref. 16	Thick plate FEM Ref. 8	Present
0	0.40624	0.41197	0.40628
1	0.40517	0.41088	0.40521
3	0.33472	0.33855	0.33473
5	0.15060	0.15114	0.15064
10	0.01115	0.10960	0.01118

Table 2. Central deflection and bending moments of SSSS square plate under uniformly distributed load (h/a=0.005;ν=0.25; K=200)

G_F	$M/ (qa^2 /100)$		$W/ (qa^4 /100D)$	
	Ref. 5	Present	Ref. 5	Present
5	2.4179	2.4208	0.2264	0.2264
10	-	1.9876	-	0.1886
20	1.6129	1.6133	0.1568	0.1570

All computations are made using 18 grid points in each direction. The results obtained by the proposed method for variation of central deflection with thickness to width ratio of CCCC plates under uniformly distributed load for different Winkler parameters (b/a=1;ν=0.3) are shown in Figure 1. The results are presented for three different Winkler parameters. It is shown that the increasing value of K always decreases the deflection. It is concluded that the w values increase as the h/a ratio increase. The parameter K of the Winkler foundation has been found to have significant influence on the static response of the Mindlin plates.

The relationship between the deflections with h/a ratio for different value of G_f is shown in Fig. 2 by setting b/a=1, K=200. It can be seen that, the displacement increases directly. It is also shown that the increasing value of G_f always decreases the deflection. Furthermore, the displacements decrease with increasing the shear modulus of foundation, G_f. Figure 3 describes the manner of variation of deflections with respect to shear modulus of foundation G_f for three different boundary conditions. Results given in this figure are obtained by setting b/a=1, h/a=0.2, K=1. It is shown in these figures that the deflections decrease with increasing shear modulus of foundation G_f. Also, the SSSS plate has the highest bending moments, followed by SCSC and CCCC.

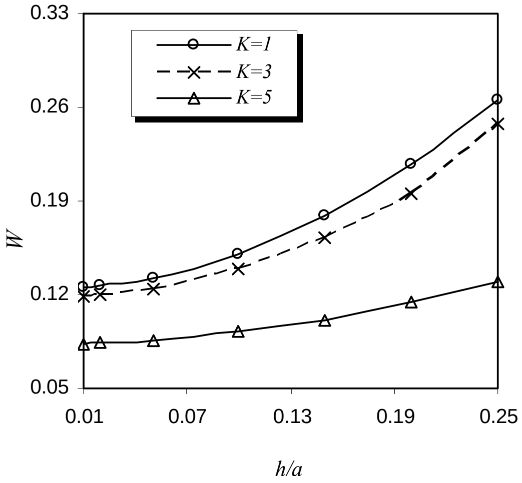

Figure 1. Variation of central deflection ($qa^4/10^4D$) with thickness to width ratio of CCCC plates under uniformly distributed load for different Winkler parameters (b/a=1;ν=0.3)

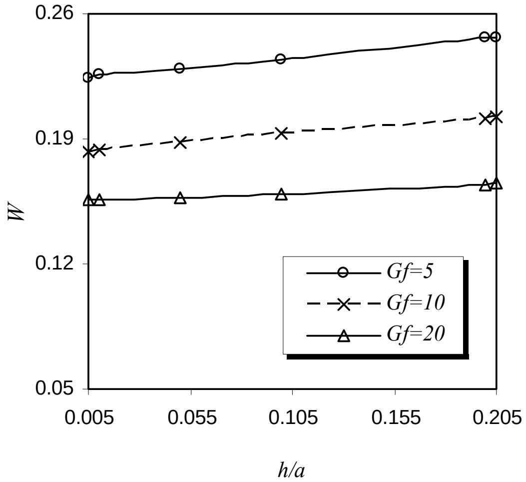

Figure 2. Variation of deflection ($qa^4/100D$) with thickness to width ratio of SSSS plates under uniformly distributed load for different Pasternak parameters (X=0.5; Y=0.5; b/a=1; K=200;ν=0.3)

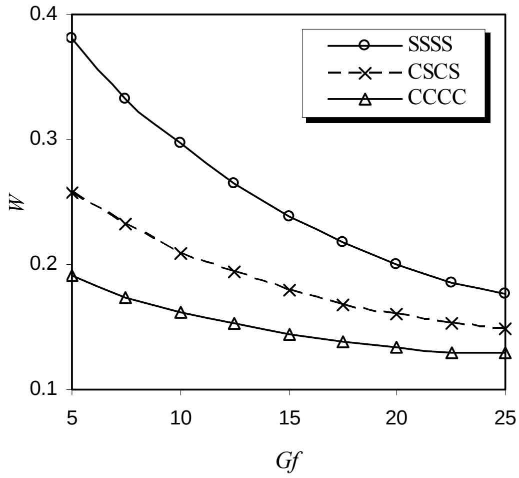

Figure 3. Variation of deflection ($qa^4/10^4D$) with Pasternak parameters under uniformly distributed load for different boundary conditions (X=0.5; Y=0.5; b/a=1; h/a=0.2; K=1;ν=0.3)

5. CONCLUSIONS

The method of discrete singular convolution for the solution of static analysis of Mindlin plates on Winkler-Pasternak foundations. The shear parameter G_f of the Pasternak foundation and stiffness parameter K of the Winkler foundation have been found to have a significant influence on the static response of the plates. Furthermore, the response to a simply supported plate is higher than the response to a clamped supported. The dimensionless deflection (W) values increase as the *h/a* ratio increase. In fact the deflection values (w) decrease.

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