

Cost Minimization Of Recycling Processes At Eti Aluminum Plants

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Received: 28 December 2011, Accepted: 12 February 2012

Abstract: This paper presents a minimization model to reduce cost of recycling process of caustic in red mud in hydrate serving product facility, a unit of ETI Alumina Plants. Caustic is very important for ETI Alumina Plants, because it is so expensive that an optimization procedure is necessary for the cost minimization. At the same plants there is also a hydrate serving product facility. One should, therefore, determine the global cheapest mixing and recycling cost. A generic non-linear program formulation for a recycling process is available in the literature, which is employed in this study. This program helps to minimize the cost of caustic recycling process.

Key words: Cost minimization, recycling, chemical processes, optimization

Eti Alüminyum Fabrikalarında Geri Dönüşüm İşlemlerinin Maliyet Minimizasyonu

Özet: Bu makale belli bir Eti Alüminyum Fabrikasındaki koston yeniden kazanımının maliyetini azaltmak için bir minimizasyon modeli sunar. Kostik, Eti Alüminyum Fabrikaları için çok önemlidir. Çünkü, geri dönüşüm maliyeti çok yüksektir ve bu maliyeti düşürmek için bir optimizasyon gereklidir. Bu yüzden, en ucuz karışım ve geri dönüşüm maliyeti belirlenmelidir. Literatürde, geri dönüşüm prosesi için bir generic non-lineer program formülasyonu mevcuttur; bu çalışmada bu non-lineer programlama probleminin formülasyonu kullanılmıştır. Bu programlama, koston geri dönüşüm maliyetini minimize eder.

Anahtar Kelimeler: Maliyet minimizasyonu, geri dönüşüm, kimyasal süreçler, optimizasyon

1. Introduction

Optimization is to choose the best solution under a set of certain conditions. There are several techniques used to solve optimization problems. These techniques are used to get the best solution. Some of them are linear programming, integer programming, quadratic programming and nonlinear programming [1].

A chemical process comprises a series of processing steps. In most chemical processes, there are a number of streams recycled back to different process units. These streams have to meet certain process requirements such as flow rate and composition [2].

As it is mentioned by [2,3], there are sources and sinks in a chemical process. A source is any stream in the process carrying a species of interest. A sink is any unit in the process and handles. A fresh source consists of fresh species. A process source has been

generated in the process. The fresh sources are obtained externally and composed of either one pure species or a mixture of different species. Their cost is dependent on the market value of the pure species present in them. Process sources are composed of more than one species. Process source' cost depends on the manner in which they are created. Due to the limitations of the process generation, there might be upper bounds on the flow rates of the process sources [2].

Zebalah et al. [4] described and used an ant colony meta-heuristic optimization method to solve the redundancy optimization problem in plastic recycling industry. This problem is known as total investment-cost minimization of series-parallel plastic recycling system. They reported that the ant colony approach had been successfully applied to the classical traveling salesman problem and it showed very good solutions in any applied area. The ant colony has also been adapted successfully to other combinatorial optimization problems. Yazici et al. [5] demonstrate that the consumption of electric-electronic equipments leads to the increase of electronic-wastes (e-wastes). These e-wastes include high content of metals and precious metals. For the recovery of metals from e-wastes, various treatment options are used based on conventional physical, hydrometallurgical, biohydrometallurgical and pyrometallurgical processes. Pons et al. [6] worked on a mathematical programming approach to optimize direct recycle-reuse networks together with wastewater treatment processes in order to satisfy a given set of environmental regulations. They developed a disjunctive programming formulation to optimize the recycle of process streams to units and the performance of wastewater treatment units. They used the MINLP model to minimize the total annual cost of the system, which includes the cost for the fresh sources, the piping cost for the process integration and the waste stream treatment cost.

2. Application Of The Recycling Problem On Eti Alumina Plants

Significant amount of caustic is used as raw material in ETI Alumina Plants. In hydrate production facility, a unit of the plant, red mud is thrown out of the system as a waste. There is significant amount of caustic in this waste red mud, which raises production cost. The plants have a recycling facility for red mud to minimize the lost. Plenty of Na_2O and Al_2O_3 compositions in red mud need to be recovered via recycling processes. The aim is to gain caustic with the most concentration at the minimum cost. The stream rates need reaching the minimum value to attain this objective. The stream schema of the recycling facility in the plant is given in Figure 1.

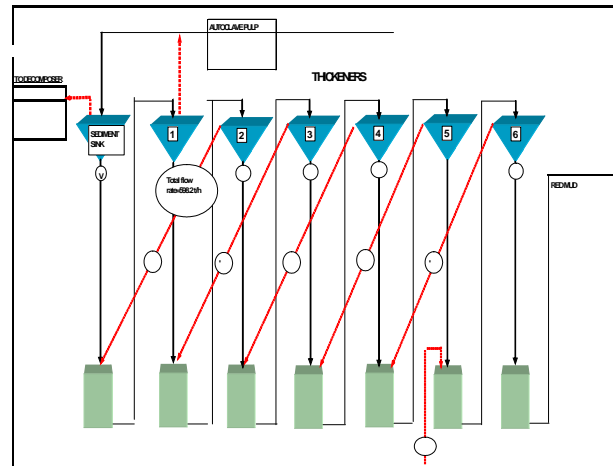


Figure 1. The stream schema of the recycling facility in the plant (The schema is taken from ETI Alumina Plants).

One can define the problem in accordance with [2] as follows. The

system consists of one process source called as red mud, another one in sink process called as the sediment sink and two other fresh sources consisting of species Na_2O and Al_2O_3 . Let us define those fresh sources as source 1 and source 2, respectively, and define process source as source 3. In such a case the following quantities as variables, and parameters are necessary for the optimization process.

C_1 : Unit cost of source 1 (TL/ton).

C_2 : Unit cost of source 2 (TL/ton).

C_3 : Unit cost of source 3 (TL/ton).

L_1 : Total flow rate of source 1(ton/hrs).

L_2 : Total flow rate of source 2 (ton/hrs).

L_3 : Total flow rate of source 3 (ton/hrs).

L_1^{sink} : Total inlet flow rate of detritus sink (ton/hrs).

$l_{1,1}$: Individual flow rate from source 1 to the sediment sink (ton/hrs).

$l_{2,1}$: Individual flow rate from source 2 to the sediment sink (ton/hrs).

$l_{3,1}$: Individual flow rate from source 3 to the sediment sink (ton/hrs).

$N1_1$: Number of iterations for total sink inlet flow rate of the sediment sink.

$N2_{1,1}$: Number of iterations for composition of Na_2O in the sediment sink.

$N2_{1,2}$: Number of iterations for composition of Al_2O_3 in the sediment sink.

$N3_1$: Number of iterations for flow rate of source 1.

$N3_2$: Number of iterations for flow rate of source 2,

$N3_3$: Number of iterations for flow rate of source 3,

$N4_{3,1}$: Number of iterations for composition of Na_2O species of source 3.

$N4_{3,2}$: Number of iterations for composition of Al_2O_3 species of source 3.

N_{large} : Arbitrarily selected large number.

N_{sink} : Total number of sinks.

N_{source} : Total number of sources.

N_{species} : Total number of species.

t_1 : Iteration index corresponding to total sink inlet flow rate of the sediment sink.

$u_{1,1}$: Iteration index corresponding to composition of Na_2O species in the sediment sink.

$u_{1,2}$: Iteration index corresponding to composition of Al_2O_3 species in the sediment sink.

v_1 : Iteration index corresponding to flow rate of source 1.

v_2 : Iteration index corresponding to flow rate of source 2.

v_3 : Iteration index corresponding to flow rate of source 3.

$w_{3,1}$: Iteration index corresponding to composition of Na_2O species of source 3.

$w_{3,2}$: Iteration index corresponding to composition of Al_2O_3 species of source 3.

$x_{3,1}$: Composition of Na_2O species in source 3.

$x_{3,2}$: Composition of Al_2O_3 species in source 3.

$z_{1,1}^{\text{sink}}$: Inlet composition of Na_2O species for the sediment sink.

$z_{1,2}^{\text{sink}}$: Inlet composition of Al_2O_3 species for the sediment sink.

r_3 : Known upper bound on flow rate of source 3.

S_1 : Known inlet flow rate of the sediment sink.

$t_{1,1}$: Known inlet mass load of Na_2O species to the sediment sink.

$t_{1,2}$: Known inlet mass load of Al_2O_3 species to the sediment sink.

The problem definition will have the following steps.

Objective function:

Since the aim is to minimize the cost one should reduce stream rates while minimizing the cost, due to the more stream rate occurrences, and the more compositions in the sources that participate in the process, thus, the more amount of material can be reduced.

$$\min = L_1C_1 + L_2C_2 + L_3C_3.$$

Source material balance:

For process sources, there is no upper bound on flow rate. Thus, it is written as an equation constraint. However, the upper bound on flow rate is known for the process sources and represented through the following conditions and restriction.

$$\begin{aligned} l_{1,1} &= L_1, \\ l_{2,1} &= L_2, \\ l_{3,1} &\leq r_3. \end{aligned}$$

Sink material balance:

$$l_{1,1} + l_{1,2} + l_{1,3} = L_1^{\text{sink}}.$$

Sink inlet composition balance:

$$\begin{aligned} l_{1,1}x_{1,1} + l_{2,1}x_{2,1} + l_{3,1}x_{3,1} &= t_{1,1}, \\ l_{1,1}x_{1,2} + l_{2,1}x_{2,2} + l_{3,1}x_{3,2} &= t_{1,2}. \end{aligned}$$

Source limits:

$$\begin{aligned} x_{1,1, \text{lower}} &\leq x_{1,1} \leq x_{1,1, \text{upper}}, \\ x_{1,2, \text{lower}} &\leq x_{1,2} \leq x_{1,2, \text{upper}}, \\ x_{2,1, \text{lower}} &\leq x_{2,1} \leq x_{2,1, \text{upper}}, \\ x_{2,2, \text{lower}} &\leq x_{2,2} \leq x_{2,2, \text{upper}}, \\ L_{3, \text{lower}} &\leq L_3 \leq L_{3, \text{upper}}. \end{aligned}$$

Sink inlet limits:

$$z_{j,k, \text{lower}}^{\text{sink}} \leq z_{j,k}^{\text{sink}} \leq z_{j,k, \text{upper}}^{\text{sink}}, \quad j = 1, 2, \dots, N_{\text{sink}} \text{ and } k = 1, 2, \dots, N_{\text{species}},$$

$$L_{j, \text{upper}}^{\text{sink}} \leq L_j^{\text{sink}} \leq L_{j, \text{lower}}^{\text{sink}}, \quad j = 1, 2, \dots, N_{\text{sink}} \text{ and } k = 1, 2, \dots, N_{\text{species}}.$$

After the general definition of the problem as above, a solution strategy is adapted from [2]. Accordingly one can write the following equations;

$$\Delta L_1^{\text{sink}} = \frac{L_{1, \text{upper}}^{\text{sink}} - L_{1, \text{lower}}^{\text{sink}}}{N1_1}, \quad (1)$$

$$\Delta z_{1,1}^{\text{sink}} = \frac{z_{1,1, \text{upper}}^{\text{sink}} - z_{1,1, \text{lower}}^{\text{sink}}}{N2_{1,1}}, \quad (2)$$

$$\Delta z_{1,2}^{\text{sink}} = \frac{z_{1,2, \text{upper}}^{\text{sink}} - z_{1,2, \text{lower}}^{\text{sink}}}{N2_{1,2}},$$

$$\Delta L_3 = \frac{L_{3, \text{upper}} - L_{3, \text{lower}}}{N3_3}, \quad (3)$$

$$\Delta x_{3,1} = \frac{x_{3,1, \text{upper}} - x_{3,1, \text{lower}}}{N4_{3,1}}, \quad (4)$$

$$\Delta x_{3,2} = \frac{x_{3,2, \text{upper}} - x_{3,2, \text{lower}}}{N4_{3,2}}.$$

Furthermore, equations (1), (2), (3) and (4) can also be written as follows.

$$L_1^{\text{sink}} = L_{1, \text{upper}}^{\text{sink}} - t_1 \Delta L_1^{\text{sink}},$$

$$z_{1,1}^{\text{sink}} = z_{1,1, \text{upper}}^{\text{sink}} - u_{1,1} \Delta z_{1,1}^{\text{sink}},$$

$$z_{1,2}^{\text{sink}} = z_{1,2, \text{upper}}^{\text{sink}} - u_{1,2} \Delta z_{1,2}^{\text{sink}},$$

$$L_3 = L_{3, \text{upper}} - v_3 \Delta L_3,$$

$$x_{3,1} = x_{3,1, \text{upper}} - w_{3,1} \Delta x_{3,1},$$

$$x_{3,2} = x_{3,2, \text{upper}} - w_{3,2} \Delta x_{3,2},$$

In mixture problems, all compositions should be equal to 1.0 [7]. So one can write the equations (5) and (6) as

$$z_{1,1}^{\text{sink}} + z_{1,2}^{\text{sink}} = 1 \quad (5)$$

$$x_{1,1} + x_{1,2} = 1 \quad (6)$$

In the problem solution a loop is necessary that takes into consideration the increasing values of $t_1, u_{1,1}, u_{1,2}, v_3, w_{3,1}$ and $w_{3,2}$. Each of iteration in the loop is applied until the following conditions are satisfied.

$$t_1 \leq N1_1$$

$$u_{1,1} \leq N2_{1,1}$$

$$u_{1,2} \leq N2_{1,2}$$

$$v_3 \leq N3_3$$

$$w_{3,1} \leq N4_{3,1}$$

$$w_{3,2} \leq N4_{3,2}$$

Minimum cost can be found according to arbitrary numbers of iteration by an already mentioned algorithm due to [2]. This solution can change according to increasing numbers of iterations and it may not give the true solutions every time. This matter can be overcome by the developed stream schema given in Figure 2. This schema can be used while solving the problem by using the Maple program.

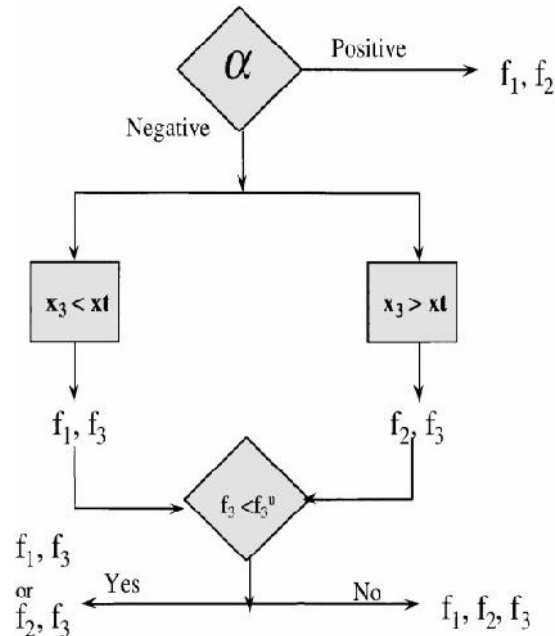


Figure 2. A stream schema for two fresh sources and one process sources.

This stream schema is for minimization of cost of recycling by using the simplex method. The relational cost parameter (r) is first calculated according to

$$r = (C_3 - C_2) + x_3(C_2 - C_1).$$

If r is positive, then the cheapest option is to use f_1 and f_2 as

$$f_1 = ft * xt,$$

$$f_2 = ft * (1 - xt).$$

On the other hand, if r is negative, then there are two possibilities [2].

Case 1. $x_3 < xt$,

$$f_1 = \frac{ft * (xt - x_3)}{(1 - x_3)},$$

$$f_2 = 0,$$

$$f_3 = \frac{ft * (1 - xt)}{(1 - x_3)}.$$

Case 2. $x_3 > xt$,

$$f_1 = 0,$$

$$f_2 = \frac{ft * (x_3 - xt)}{x_3},$$

$$f_3 = \frac{ft * xt}{x_3}.$$

If the calculated f_3 value exceeds f_3^u , one has to use all three sources to get the cheapest solution.

$$f_1 = ft * xt - x_3 * f_3^u,$$

$$f_2 = ft * (1 - xt) - (1 - x_3) * f_3^u,$$

$$f_3 = f_3^u.$$

3. Findings

Let us define the symbols given above and write the known values as follows.

C_1 : Unit cost of source 1 (TL/ton)

C_2 : Unit cost of source 2 (TL/ton)

C_3 : Unit cost of source 3 (TL/ton).

x_3 : Composition of source 3.

xt : Total composition of sediment the sink.

f_3^u : Known upper bound on flow rate of source 3.

ft : Total inlet flow rate of sediment sink (ton/hrs)

f_1 : Total flow rate of source 1 (ton/hrs)

f_2 : Total flow rate of source 2 (ton/hrs)

f_3 : Total flow rate of source 3 (ton/hrs)

The known values are given in Table.1.

Table 1. The values taken from ETI ALUMINA PLANTS

$C_1=14.17$ TL/ton
$C_2=76.7$ TL/ton
$C_3=30$ TL/ton
$x_3=598.2$ ton/hrs
$xt=0.445$
$f_3^u=150$ ton/hrs

One can solve this problem by using the following Maple codes.

```
> restart;
```

```
> f3u := 150 : x3 := 0.288 : c1 := 14.17 : c2 := 76.7 : c3 := 30 : ft := 598.2 : xt := 0.445 :
> alpha := (c3 - c2) + x3 * (c2 - c1);
```

```
> if alpha > 0 then
```

```
  f1 := ft * xt;
```

```
  f2 := ft * (1 - xt);
```

```
  cost := f1 * c1 + f2 * c2;
```

```
elif alpha < 0 and x3 < xt then
```

```
  f1 := ft * (xt - x3) / (1 - x3);
```

```
  f2 := 0;
```

```
  f3 := ft * (1 - xt) / (1 - x3);
```

```
else
```

```
  f1 := 0; f2 := ft * (x3 - xt) / x3; f3 := ft * xt / x3; cost := f1 * c1 + f2 * c2 + f3 * c3;
```

```
end if;
```

```
> if f3 > f3u then  
f1 := ft * xt - x3 * f3u;  
f2 := ft * (1 - xt) - (1 - x3) * f3u;  
f3 := f3u;  
cost := f1 * c1 + f2 * c2 + f3 * c3; end if;
```

4. Conclusion

The results of this study can be summarized as follows.

$$f_1 = 222.9990 \text{ ton/h,}$$

$$f_2 = 225.2010 \text{ ton/h,}$$

$$f_3 = 150 \text{ ton/h,}$$

Minimum recycling cost is 24932.81 TL/ton, which means that the system should work with that flow rate to achieve the minimum cost. If one compares the solution with the cost of the recycling processes in ETI Alumina Plants, then he/she can see that it is much cheaper than the cost spent.

The well-known basic simplex method involves choosing $n+1$ points, where n is the number of variables. The main rule for the simplex evolution is to eliminate the worst point and to replace it by its symmetrical with respect to the centroid of the hyperface formed by the remaining simplex points. One could solve the present problem by using the simplex method, too. However, it is more complex and takes much time to solve. By using this algorithm, one can reach the results more speedily.

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